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THESIS

EVALUATIONS OF APPROXIMATIONS FOR SHORTAGE COSTS FROM MAJOR CUSTOMERS FOR A SINGLE PERIOD INVENTORY MODEL

by

Wilhelm F. Noeggerath

September 1984

Thesis Advisor: Alan W. McMasters

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Evaluations of Approximations for Shortage Costs from Major Customers for a Single Period Inventory Model

by

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Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

A. GENERAL CHARACTERISTICS OF INVENTORY MODELS

Inventory models have been developed and used to determine the optimal policy for running inventory systems since 1915. The objective is usually to minimize the annual expected variable costs associated with running the inventory system. Usually the costs are aggregated under three cost categories:

1. The order costs depend on the frequency of orders for resupply.
2. The holding costs are those costs associated with carrying stock in the inventory system.
3. The shortage costs are those costs which are incurred whenever the system runs out of stock.

When a specific model is constructed and applied to a real world situation two additional goals tend to be sought. The first objective is to reflect within the model the relevant features of the specific type of real world situation to a satisfying degree. The second objective is to limit the costs associated with applying the inventory model. These costs consist mainly of costs of data collection, data transfer,and computation. In general, a better result in attaining one goal implies a reduction in fulfilment of the other goal.

This thesis will consider that special class of inventory models known as the single period model. Such a model attempts to determine an optimal quantity of an item to buy and stock which will minimize the holding and shortage costs over the period's duration. Order costs are usually ignored because an order is assumed to be placed at the start of every given period.

B. DETERMINING THE SHORTAGE COST

Of the three cost categories the shortage costs are usually the most difficult to express in dollar values because the data needed to evaluate such costs is seldom available in a readily useable form. It typically is included as part of overhead costs or indirect costs.

Assuming that shortage costs can be quantified, the usual approach is to associate the event of 'no delivery' with a fixed number of dollars for each item not delivered immediately upon request. This can be refined by adding a time-dependent dollar cost which is a linear function of the length of the time period between the occurrence of the shortage and the end of the period. An implicit assumption is that the customer population is homogeneous.

This thesis considers the case where the customer population is not homogeneous. In this case several different major customers are assumed and there is a different demand distribution and fixed unit shortage cost for each major customer. The unit shortage cost is assumed to be constant for each customer.

C. THE NARF MODEL

The specific model considered in this thesis is the single period model presented by McMasters [Ref. 1]. That model is intended to provide supply support for overhauls of repairable aircraft systems done by Naval Air Rework Facilities (NARF's). The following assumptions apply:

1. Within the modelled single period, repair parts are needed for the overhaul of different end items. Each end item can be viewed as a different major customer.

During the repair of an end item, a demand for a given part occurs with probability p, where p is greater than zero ,(and no demand will occur with probability (1-p)). Demands are satisfied on a "first-come, first-served" basis.

2. The repair cycles for each type of end item are nonoverlapping and one cycle immediately follows its predecessor. There may be a different number of repair cycles per period for different end items.
3. A fixed unit shortage cost is associated with a specific end item. This unit shortage cost may be different for different end items.
4. The quantity to buy for inventory is computed and placed in stock at the beginning of each quarter (period) and no replenishment during the quarter is allowed. It is assumed that the supporting Naval Supply Center (NSC) maintains the stock or will obtain it by the beginning of the quarter when it is needed.

Within this setting the expected shortage costs for the period are a function only of:

1. The customer specific fixed unit shortage cost;
2. The order in which demands occur during the period;
3. The initial stock available to satisfy demand.

When a demand occurs, the amount demanded is one unit and the order in which each demand occurs depends on the probability distribution of the inter-demand times for each customer, where the inter-demand time is the time between successive occurrences of demand by that customer. The total demand by each customer during the period is binomially distributed and is assumed to be stochastically independent of the demand of any other customer.

McMasters suggested an approximation formula for the expected shortage cost per period (two customers) [Ref. 1].

$$\text{expected shortage cost per period} = \frac{\pi_1\mu_1 + \pi_2\mu_2}{\mu_1 + \mu_2}\int_Y^\infty [x-Y]\,f(x)\,dx$$

or, for discrete demands,

$$\frac{\pi_1\mu_1 + \pi_2\mu_2}{\mu_1 + \mu_2}\sum_{x=Y+1}^{\infty}(x-Y)P(x)$$

where

Y = purchased quantity, initially available for delivery ;

$x = x_1 + x_2$;

x_1 = demand per period, customer 1 ;

x_2 = demand per period, customer 2 ;

μ_1 = mean demand per period for customer 1 ;

μ_2 = mean demand per period for customer 2 ;

π_1 = unit shortage cost for customer 1 ;

π_2 = unit shortage cost for customer 2 ;

$f(x)$ = density function for x.

$P(x)$ = probability mass function for x

The first term attempts to approximate the average unit shortage cost. The second term evaluates the expected number of shortages per period. The first term represents a convex combination of π_1 and π_2 weighted by μ_1 and μ_2. Its range of values is bounded by π_1 and π_2. A large value of μ_1 drives the value of the first term towards π_1 whereas a large value of μ_2 drives it towards π_2.

Several theses have expanded the work of McMasters. Slaybaugh [Ref. 2] analyzed the demand data and the rework schedule during 1980 for the TF-34 aircraft engine being reworked by NARF at the Naval Air Station Alameda, California. Combining those data with the engine's bill of materials, he was able to estimate the probability of replacement per repair action and conducted a parametric analyses to study the relationship between the shortage and surplus costs.

Asselin [Ref. 3] applied dynamic programming in order to expand the single period model to a multi-period model. The development of a "good" formula for the expected shortage costs per period in the single period model is a basic building block for an efficient multi-period model.

D. PURPOSE

The purpose of this thesis is to evaluate the approximation for the shortage cost term suggested by McMasters and to investigate alternatives to that approximation.

E. PREVIEW OF ANALYSIS

Chapter II shows that McMasters' approximation formula provides the exact expected shortage costs per period in case of an arbitrary number of mutually independent Poisson distributed customer demands. Because McMasters' approximation formula does not provide the exact expected shortage costs per period in case of binomial distributed customer demands, other approximation formulas are presented in Chapter III.

Simulation is then used in Chapter IV to VI to evaluate the performance of the different approximation formulas. Chapter IV describes the simulation methodology. Chapter V contains simulation results and discussion for the case of two customers each with binomially distributed demands. Chapter VI expands the analysis to more than two customers. The normal distribution is used to approximate the sum of the binomial demand distributions and McMasters' approximation formula is applied. This procedure is then analyzed by running simulations.

A final summary with conclusions and recommendations is presented in Chapter VII.

II. POISSON DISTRIBUTED CUSTOMER DEMANDS

A. PRELIMINARY REMARKS

Suppose that demands for each customer can be described by independent Poisson processes. In general, the Poisson distribution is often used to model infrequent random events. For instance, the UICP inventory model of the US Navy for consumable items assumes the very low demand items to be Poisson distributed (an average demand of 1 unit or less per year).

If the Poisson assumption is valid then the expected shortage costs per period are exactly equal to McMasters' approximation and can be easily derived using conditional probability arguments. Because McMasters' approximation formula basically multiplies an average of the unit shortage costs by the expected number of shortages per period it will be named 'monovariable approximation formula' (i.e. only the sum varies).

B. COMPUTATION OF CONDITIONAL EXPECTED VALUE

Ross [Ref. 4 pp.75-76] determines the conditional expected value of X_1 given $X_1 + \dots + X_n = x$, where $X_1, \dots, X_n$ are independent Poisson random variables with means $\mu_1, \dots, \mu_n$.[1]

First the conditional probability that $X_1 = k$ given $X_1 + \dots + X_n = x$ is calculated:

---

[1] Ross computes the conditional expected value for the case of two Poisson distributions. His path of reasoning is applied here to compute the conditional expected value in case of n Poisson distributions for any $n \geq 2$.

$$P\{X_1=k \mid X_1+\cdots+X_n\} = \frac{P\{X_1=k,\ X_1+\cdots+X_n = x\}}{P\{X_1+\cdots+X_n = x\}}$$

$$= \frac{P\{X_1=k,\ X_2+\cdots+X_n = x-k\}}{P\{X_1+\cdots+X_n = x\}}$$

$$= \frac{P\{X_1=k\}\,P\{X_2+\cdots+X_n = x-k\}}{P\{X_1+\cdots+X_n = x\}}$$

Substituting the appropriate probability mass functions yields:

$$P\{X_1=k \mid X_1+\cdots+X_n = x\}$$

$$= \frac{e^{-\mu_1}\mu_1^k}{k!}\ \frac{e^{-(\mu_2+\cdots+\mu_n)}(\mu_2+\cdots+\mu_n)^{(x-k)}}{(x-k)!}\left[\frac{e^{-(\mu_1+\cdots+\mu_n)}(\mu_1+\cdots+\mu_n)^x}{x!}\right]^{-1}$$

$$= \binom{x}{k}\frac{\mu_1^k\,(\mu_2+\cdots+\mu_n)^{(x-k)}}{(\mu_1+\cdots+\mu_n)^x}$$

$$= \binom{x}{k}\frac{\mu_1^k}{(\mu_1+\cdots\mu_n)^k}\ \frac{(\mu_2+\cdots+\mu_n)^{(x-k)}}{(\mu_1+\cdots+\mu_n)^x(\mu_1+\cdots+\mu_n)^{(-k)}}$$

$$= \binom{x}{k}\left(\frac{\mu_1}{\mu_1+\cdots+\mu_n}\right)^k\left(\frac{\mu_2+\cdots+\mu_n}{\mu_1+\cdots+\mu_n}\right)^{(x-k)}$$

Therefore the conditional distribution of X_1 given $X_1 + \ldots + X_n = x$ is the binomial distribution with parameters x and $\frac{\mu_1}{\mu_1+\cdots+\mu_n}$. By symmetry it follows that the conditional distribution of X_i given $X_1 + \ldots + X_n = x$ is the binomial distribution with parameters x and $\frac{\mu_i}{\mu_1+\cdots+\mu_n}$ for i=1, ... , n. The associated conditional expected values are :

$$E[\,X_i \mid X_1+\ldots+X_n = x] = x\,\frac{\mu_i}{\mu_1+\ldots+\mu_n}\ . \qquad (2.1)$$

C. DERIVATION OF THE FORMULA FOR EXPECTED SHORTAGE COSTS PER PERIOD

Given a shortage occurs, the demand generating the stockout is due to customer i with probability $\frac{\mu_i}{\Sigma\mu}$ and the penalty is π_i dollars. The expected penalty cost for the stockout is therefore

$$\sum_{i=1}^{n} \pi_i \frac{\mu_i}{\mu_1 + \ldots + \mu_n} .$$

Now the number of stockouts given x demands in the period is

$$\begin{cases} 0 & \text{if } x \le Y \\ x-Y & \text{if } x > Y \end{cases} .$$

The expected number of stockouts is therefore,

$$\sum_{x=Y+1}^{\infty} (x-Y)\ P(x),$$

where P(X) denotes the probability mass function of X.

Each of these stockouts results in an expected cost of

$$\bar{\pi} = \frac{\Sigma \pi_i \mu_i}{\Sigma \mu_i} .$$

Therefore the total expected shortage costs per period can be written as:

$$\bar{\pi} \sum_{x=Y+1}^{\infty} (x-Y)\ P(x) .$$

The last expression is exactly the discrete analog to the approximation formula derived by McMasters [Ref. 1] expanded to an arbitrary number of customer demand distributions.

D. SUMMARY AND CONCLUSIONS

This chapter has shown that the monovarible approximation formula provides a numerically exact value for the expected shortage cost when the aggregate demand process is the sum of arbitrary number of mutually independent Poisson processes. This formula utilizes the closure theorem for the Poisson distribution, which says that the sum of independent Poisson random variables is again a Poisson random variable.

Feller [Ref. 5 p.142] shows that the binomial distribution with parameters (n,p) converges to the Poisson distribution with parameter $\mu = np$ as $n \to \infty$ and $p \to 0$ in such a way that $\mu = np$ remains bounded. So the Poisson distribution corresponds to a binomial distribution with large n and small p.

Since the Poisson is a limit of binomials and since the formula for expected stockout cost is exact for the Poisson case, it seems reasonable that the formula might also be valid for the case in which demands from each customer during a period are binomially distributed. Unfortunately, a single counterexample shows that the shortage cost expression is not exact for every case in which the demands are binomial. To see this, consider the case in which there are two customers as follows:

customer	#Repair cycles per period	Probability of demand per cycle	stockout cost
1	1	0.9	\$ 100
2	2	0.1	\$ 1,000

We will show that expression (2.1) is not valid for this example.

Let X_i be the total demand in the period by customer i. Suppose $X = X_1 + X_2 = 2$. Then

$$P(X_1+X_2=2) = \sum_{k=0}^{1} P(X_1+X_2=2 \mid X_1=k)\,P(X_1=k)$$

$$= \sum_{k=0}^{1} P(X_2=2-k)\,P(X_1=k)$$

$$= P(X_2=2)\,P(X_1=0) + P(X_2=1)\,P(X_1=1)$$

$$= p_2^2(1-p_1) + \binom{2}{1}p_2(1-p_2)p_1$$

$$= 0.1630$$

Now, the conditional probabilities are given by:

$$P(X_1=0 \mid X=2) = \frac{P(X_1=0,\ X_2=2)}{P(X=2)}$$

$$= \frac{P(X_1=0)\,P(X_2=2)}{P(X=2)} = \frac{(1-p_1)p_2^2}{0.1630} = 0.0061$$

$$P(X_1=1 \mid X=2) = \frac{P(X_1=1)\,P(X_2=1)}{P(X=2)} = \frac{\binom{2}{1}p_2(1-p_2)p_1}{0.1630} = 0.9939$$

and, the conditional expectation is:

$$E[X_1 \mid X=2] = 0P(X_1=0 \mid X=2) + 1P(X_1=1 \mid X=2) = 0.9939 \qquad (2.2)$$

Now let us compare (2.2) to the value given by expression (2.1); $\mu_i = n_i p_i$, and $E[X_1 \mid X=2] = 2\left(\frac{1(0.9)}{1(0.9)+2(0.1)}\right) =$ 1.6364. This is significantly larger than the exact value (2.2).

Thus we can see from the simple example that, for small n and appropriate choices of the p_i's, expression (2.1) provides a poor approximation for $E[X_i \mid X=x]$ when the total demand in a period from each customer is binomially distributed. Consequently, the monovariable expression for the total expected shortage cost will also be in error for this case. However, this example is extreme, and it might be the case that for some sets of values for the parameters the monovariable expression provides a reasonable approximation for the true shortage cost. We evaluate this possibility in a later chapter by simulating actual demands and shortages and comparing the monovariable expression to the average stockout costs computed over a large number of replications.

Before we proceed to the development of alternative approximation formulas and to the simulation results, let us first return to the simple example to understand another problem that results from the binomial demand situation. Suppose there is a single spare part to support both customers; i.e. Y=1. As above, suppose that X=2. Now, what is the shortage cost? This cost is either $ 100 or $ 1,000 depending on the sequence in which the demands occurred during the quarter. Below are three possibilities for which X=2 (S corresponds to a demand during the period and F corresponds to no demand).

```
                        S                                  F
Case 1:  1 |-----------|          Case 3:  2 |-----------|

              S     F                           S     S
         2 |-----|-----|                     |-----|-----|


                        S
Case 2:  1 |-----------|

              F     S
         2 |-----|-----|
```

In case 3, the stockout cost is clearly $ 1,000. However, in cases 1 and 2 the stockout cost could be either $ 100 or $ 1,000 depending on when during a repair cycle that the demands occurred. If, for example, it is assumed that demands occur at the end of a repair cycle, the stockout cost in case 1 is $ 100, but in case 2 it would be either $ 100 or $ 1,000. If demands are assumed to occur at the beginning of a repair cycle, the stockout cost in case 2 would be $ 1,000, but in case 1 it would be either $ 100 or $ 1,000 depending on what kind of rule is used to determine which customer gets an item when two or more demands occur simultaneously.

We see, from this example, that the timing of the demands plays a key role in the determination of the shortage costs. We examine this issue further in our simulation evaluation in the following chapters.

III. DERIVATION OF OTHER APPROXIMATION FORMULAS

A. INTRODUCTION

Before simulations are introduced in the next chapter to evaluate the accuracy of the monovariable formula for the binomial "real world" NARF situation, two further approximation formulas will be derived. Their accuracy will be evaluated along with the monovariable formula.

B. DERIVATION OF THE HYPERGEOMETRIC APPROXIMATION FORMULA

Given two discrete mutually independent demand distributions $P_1(X_1)$ and $P_2(X_2)$ the probability for each possible demand vector $<x_1,x_2>$ can be evaluated from the product of their probability mass functions, $P_1(x_1)P_2(x_2)$.

Next in the conditional setting, i.e. given a realization $<x_1,x_2>$ with $(x_1+x_2) > Y$ an interpretation of random drawings without replacements suggests an approximation based on the hypergeometric distribution. Let

x = number of balls in urn $(=x_1+x_2)$,
x_1 = number of balls with associated unit shortage cost π_1,
x_2 = number of balls with associated unit shortage cost π_2,
s = number of shortages $\{= (x_1+x_2) - y\}$.

If s balls are drawn out of an urn containing x balls, x_1 of which have associated unit shortage cost π_1 and x_2 of which have associated unit shortage cost π_2 then, according to the formula for the expected value of the hypergeometric distribution, the expected number of x_1 balls in the sample is

$$s \frac{x_1}{x_1 + x_2} ,$$

and the expected number of x_2 balls is

$$s \frac{x_2}{x_1 + x_2} .$$

Multiplying each term by the associated unit shortage cost and summing them yields:

$$s \frac{x_1}{x_1 + x_2} \pi_1 + s \frac{x_2}{x_1 + x_2} \pi_2 . \qquad (3.1)$$

This expression represents an approximation for the expected shortage costs per period given x_1, x_2, Y and the constraint that $x_1+x_2>Y$.

Because X_1 and X_2 are mutually independent random variables, the unconditional approximation for the expected shortage costs per period can be computed by multiplying each conditioned approximation with its associated probability of occurrence and summing these products, i.e.:

$$\sum_{x_1=0}^{n_1} \sum_{x_2=0}^{n_2} \left\{ max(0,(x_1+x_2)-Y) \left(\frac{x_1}{x_1+x_2} \pi_1 + \frac{x_2}{x_1+x_2} \pi_2 \right) P_1(x_1) P_2(x_2) \right\} \qquad (3.2)$$

This sum gives an approximation for the expected shortage cost per period. Unfortunately, it does not reduce to a simple formula. The hypergeometric formula is programmed as the subroutine HYPGEO in the program provided as Appendix A. This formula corresponds to the first step of McMasters' development of his approximation formula [Ref. 1].

C. DERIVATION OF THE CDF APPROXIMATION FORMULA

In the conditional setting the hypergeometric approximation formula does not reflect any information provided by the individual demand distributions. To incorporate this information, an additional weight can be introduced into the averaging operation in equation (3.1).

If $(x_1+x_2) > Y$, the quantity Y is split into two subquantities Y_1 and Y_2 such that $x_1/x_2 = Y_1/Y_2$. Suppose next that the density functions of X_1 and X_2 look like Figure 1. The areas represent the average height of the density functions in the indicated regions.

In Figure 1 a relatively small fraction of the probability mass over the region $[0,x_1]$ is located in the region $(Y_1,x_1]$ whereas a relatively large fraction of the probability mass over the region $[0,x_2]$ is located in the region $(Y_2,x_2]$.

The motivation for the CDF approximation formula is that the low fraction of probability over $(Y_1,x_1]$ would result in fewer shortages for customer 1 and the high fraction over $(Y_2,x_2]$ would result in more shortages for customer 2. Those fractions of probabilities over the regions $(Y_1,x_1]$ and $(Y_2,x_2]$ are incorporated as additional weights into the hypergeometric approximation formula.

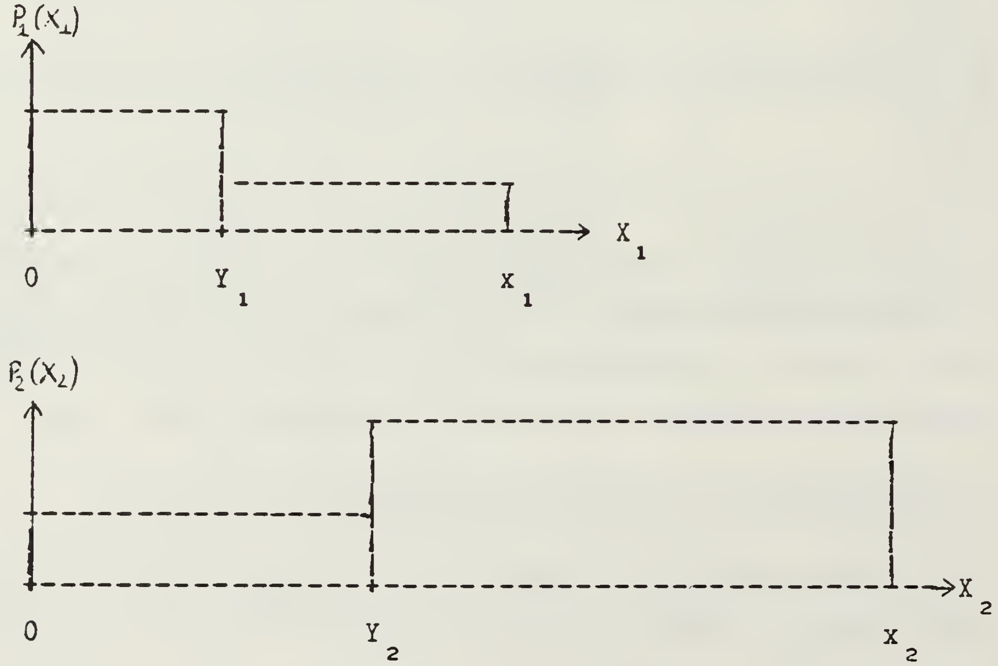

Figure 1: Possible Density Functions for X_1 and X_2.

Let CDF1 and CDF2 denote the cumulative distribution functions for X_1 and X_2 respectively. Then the additional weights f_1 and f_2 can be computed by

$$f_1 = \frac{CDF1(x_1) - CDF1(Y_1)}{CDF1(x_1)} ;$$

$$f_2 = \frac{CDF2(x_2) - CDF2(Y_2)}{CDF2(x_2)} .$$

These weights are then introduced as multipliers in the averaging operation of the hypergeometric approximation formula, i.e. equation (3.1) becomes

$$s \frac{x_1}{x_1 + x_2} \pi_1 f_1 + s \frac{x_2}{x_1 + x_2} \pi_2 f_2 . \qquad (3.3)$$

The expansion to the unconditioned setting is identical to the procedure applied when deriving the hypergeometric formula at the end of the preceding section (i.e. Equation (3.2)). The CDF approximation formula is programmed as the subroutine SUBCDF in the program provided as Appendix A.

D. COMMENTS ON THE TWO DERIVED APPROXIMATION FORMULAS

In case of the monovariable formula there is only one convex combination of the unit shortage costs computed, whereas by applying the hypergeometric or CDF formulas a convex combination of the unit shortage costs for each possible demand vector must be computed. It might be that computing the convex combinations within the conditioned setting provides better approximations than the monovariable formula.

In comparing these approximations under the Poisson distribution of Chapter II, the monovariable formula and the hypergeometric formula provide the same numerical results in the cases of mutually independent Poisson and binomially distributed demands when the latter is such that $p_i=p$ for all i.

The suggested CDF formula attempts to refine the hypergeometric approximation formula. However, it requires the computation of the subquantities Y_1 and Y_2. If Y_1 and Y_2 are not integers, a interpolation of the individual cumulative distribution functions has to be performed in order to compute the weights f_1 and f_2. An alternative approach would be to round Y_1 and Y_2 to the nearest integer. Unfortunately, both approaches cause distortions, therefore the CDF formula was not evaluated by running simulations in the Poisson situation. However, it is intuitively appealing and should be evaluated along with the monovariable and hypergeometric approximation formulas. The main question to be answered as far as the CDF approximation formula is concerned, is whether the incorporation of the distributions into the averaging operation is a step in the right direction.

E. SUMMARY

This chapter provided the arguments leading to the two further approximation formulas, called the hypergeometric approximation formula and the CDF approximation formula. Simulation is required to determine the accuracy to be expected from applying the three approximation formulas in the binomial "real world" situation of the NARF model.

IV. DESCRIPTION AND ANALYSIS OF SIMULATIONS

A. PRELIMINARY REMARKS

If the individual customer demands are binomially distributed with possibly different p_i's and n_i's there is, based upon the knowledge of the author, no closed form expression available which provides the exact value for the expected shortage costs per period. Therefore simulation was used in order to estimate the accuracy of the approximation formulas.[2] However, as with any simulation, it will provide only an estimate of the actual value of the expected shortage costs. The precision of the estimate gained from simulation is almost exclusively determined by the number of simulated periods(replications).

B. PROGRAMMING LANGUAGE AND RANDOM NUMBER GENERATOR

The simulation programs were written in FORTRAN WATFIV. WATFIV allows straightforward structured programming which improves the readability and verifiability of a program. The WATFIV compiler provides a large set of error checks and gives, in case of errors, uncoded and precise error messages. A further reason is the availability of a WATFIV specific debugging mode at the Naval Postgraduate School which eases testing of a program.

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[2] In the early stages of the research done for this thesis the author had written simulation programs for up to four customers whose demands per period were assumed to be Poisson distributed. The simulations suggested that the values of the approximation formula were in fact the exact expected shortage costs per period. Driven by this evidence the author finally succeeded in finding an analytical proof for the case of a Poisson setting.

The basic prerequisite for any Monte Carlo type of simulation is the availability of proper random numbers. The simulation programs use 'LLRANDOMII', the Naval Postgraduate School Random Number Generator Package [Ref. 6]. This package has been tested thoroughly and is well validated. It generates very good pseudo random numbers.

C. SIMULATION OUTPUT ANALYSIS FOR MEAN ESTIMATE

During simulation many independent observations (replications) of the shortage cost per period are obtained. Because, in the context of this thesis, the interest is limited to the mean of the shortage costs per period, a straightforward output analysis can be applied.[3]

The sample mean is used to estimate the mean of the shortage cost per period. It is unbiased and has minimum variance. Dividing the sample variance by the number of replications gives an unbiased estimate for the variance of the mean estimate. It represents the estimated precision of the mean estimate derived from simulation.

1. Notation

C - scalar random variable, shortage cost per period

$\bar{C}$ - sample average

n - number of replications

sd - standard deviation

Var- variance

A hat indicates an estimator, i.e. $\hat{\mathrm{Var}}(C)$ is used to estimate $\mathrm{Var}(C)$.

$$\bar{C} = \frac{1}{n} \sum_{i=1}^{n} C_i$$

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[3] The reasoning presented in this section is mainly extracted from [Ref. 7] as far as output analysis is concerned.

$$\hat{V}ar(C) = \frac{1}{n - 1} \sum_{i=1}^{n} (C_i - \bar{C})^2$$

$$\hat{V}ar(\bar{C}) = \frac{1}{n} \hat{V}ar(C)$$

$$\hat{s}d(\bar{C}) = \left(\frac{1}{n} \hat{V}ar(C) \right)^{1/2}$$

By the Central Limit Theorem the distribution of the sample average of any random variable with a well-defined mean and variance approaches more and more the normal distribution as the size of the sample becomes larger and larger. In general if there are more than twenty independent observations in a sample the normal distribution is assumed to provide a sufficient approximation. Therefore, when n is sufficiently large, the random variables

$$\frac{\bar{C} - E[C]}{sd(C)} \quad , \quad \frac{(n - 1)\ Var(C)}{\hat{V}ar(C)}$$

are distributed as a standard normal and a chi-squared distribution with (n-1) degrees of freedom, respectively.

Both statistics are independent, therefore

$$\frac{\sqrt{n}\ (\bar{C} - E[C])}{\hat{s}d(C)}$$

has a Student's t-distribution with (n-1) degrees of freedom. Because the t-distribution converges to the normal distribution with increasing n it can be assumed that

$$\frac{\bar{C} - E[C]}{\hat{s}d(C)}$$

is distributed as a standard normal random variable.

For the unknown true mean, a confidence interval can be computed with $(1-\alpha)$ as the preassigned confidence level:

$$P\{E[C] \text{ within } \bar{C} \pm Z_{\frac{\alpha}{2}} \hat{sd}(\bar{C})\} = 1-\alpha$$

where $Z_{\frac{\alpha}{2}}$ represents the standard normal deviate.

To increase the precision of the estimated mean of the shortage costs per period by a factor of two (i.e. reduce the estimated standard deviation of the mean estimate to half of its value) the number of replications has to be increased fourfold. In general, as far as mean estimation through simulation is concerned, the precision of the estimate is directly proportional to the square root of the factor by which the number of replications gets increased. All simulations run in connection with this thesis generated sample sizes far beyond one thousand.

D. OUTPUT ANALYSIS FOR SIMULATED TWO BINOMIAL DEMAND SOURCES

There are four different categories of parameters: number of repair cycles per period, probability of one demand during one repair cycle, unit shortage cost, and the purchased quantity. Each of these parameters is a candidate for causing deviations of the numerical results of the three approximation formulas from the actual shortage costs per period. The tests of all three approximation formulas were restricted to two binomial demand sources in order to more easily identify those parameters which most affect the accuracy of the approximations.

The output tableau of the simulation program and explanations of its entries are provided as Appendix F. The simulation results presented in Chapter V for the different settings display only those values from this tableau which are of intermediate interest.

V. SIMULATION RESULTS FOR TWO DEMAND SOURCES

A. PRELIMINARY REMARKS

The program to perform the simulations for two demand sources is provided as Appendix A to this thesis. The following sections of this chapter have the following structure:

1. Purpose of the chosen parameter setting;
2. Presentation of the simulation results;
3. Analysis of the simulation results.

B. FIRST SIMULATION SETTING

In the first simulation setting the number of repair cycles and the initial seed are fixed for all runs. The p's and Y vary to analyze their influence on the precision of the approximation formulas. In Tables 1-4 the unit shortage costs are \$ 100 and \$ 1,000 for demand source 1 and demand source 2, respectively. In Table 5 the unit shortage cost for source 2 was reduced from \$ 1,000 to \$ 200 in order to analyze the effect of a lower variability among the unit shortage costs on the accuracy of the approximations.

For each table the purchased quantity Y is a specific multiple of the sum of expected demands per period from both demand sources. The value of Y is always rounded to the nearest integer. However, Y was increased by at least one unit in the sequence of Tables 1-4 in order to examine the influence of a change in Y on the accuracy of the approximations.

The simulation results presented in this and the following section are arranged in tableaus consisting of blocks of six numbers which refer to specific parameter values.

shortage-cost [4]	SD/MEAN [5]	y
DEVMON [6]	DEVHYP [6]	DEVCDF [6]

The meanings and units of the abbreviations are the same as described in Chapter IV, section D. In the tableaus, P1 and P2 denote the p values for the binomial distributions for demand source 1 and demand source 2, respectively. The maximal sum of both demands per period is 9 units for Tables 1-5.

1. Parameter Setting for Tables 1-4: [7]

Number of time units	20
Number of repair cycles(1)	4
Number of repair cycles(2)	5
Unit shortage cost(1)	$ 100.-
Unit shortage cost(2)	$ 1000.-
Initial seed for all runs: [8]	150449

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[4] The observed sample mean for shortage costs per period derived from the simulation.

[5] The observed standard deviation for the sample mean divided by the sample mean provides a measure (in percent) of the precision of the simulation.

[6] Deviations of the values of the three different approximation formulas from the simulation result in percent.

[7] In order to avoid having very large values for Y in Tables 2-4 for the case of P1=0.1 and P2=0.1, Y=0 was chosen for that case in Table 1.

[8] For all simulations in this section the same initial seed was used in order to reduce the variability of the simulation results. Differences in the performance of the approximation formulas between the tables are therefore mostly due to different levels of Y.

2. Table 1: Y About 15% Below Mean Demand

P1 10,000 repl's

.9	284	2	3	672	1	5	775	1	6	772	1	7
	-16	1	-35	-15	-6	-32	-15	-11	-26	-10	-10	- 3
.7	347	1	2	813	1	4	898	1	5	979	1	6
	-8	2	-32	-7	-4	-13	-5	-5	3	-8	-11	2
.5	228	2	2	550	1	4	585	1	5	579	1	6
	-9	2	-30	-6	-6	-3	-3	-7	3	-6	-13	8
.1	527	1	0	897	1	2	914	1	3	871	1	4
	3	3	5	3	1	10	3	0	8	2	-2	7
P2	.1			.5			.7			.9		

3. Table 2: Y About Equal to Mean Demand

P1 10,000 repl's

.9	122	3	4	342	2	6	358	1	7	382	1	8
	-39	-2	-48	-26	-15	-50	-23	-18	-31	-39	-39	-42
.7	173	2	3	434	1	5	478	1	6	456	1	7
	-19	1	-35	-13	-8	-20	-12	-12	-10	-13	-18	2
.5	91	3	3	243	2	5	248	2	6	197	2	7
	-21	-1	-27	-10	-10	-11	-10	-15	-3	-11	-22	6
.1	164	3	1	372	2	3	350	2	4	229	2	5
	6	6	10	3	-2	7	3	-4	6	3	-10	4
P2	.1			.5			.7			.9		

4. Table 3: Y About 15% Above Mean Demand [9]

P1 50,000 repl's

.9	28	4	5	114	2	7	110	2	8	0	0	9
	-54	-7	-70	-38	-24	-62	-46	-40	-68	***	***	***
.7	58	3	4	184	1	6	182	1	7	140	2	8
	-35	1	-32	-23	-16	-34	-22	-22	-19	-34	-39	-21
.5	24	4	4	83	2	6	71	2	7	35	3	8
	-32	-1	-13	-19	-19	-21	-19	-26	-12	-24	-37	-2
.1	37	3	2	110	2	4	81	2	5	32	4	6
	0	0	0	-1	-9	-4	-1	-14	-7	-1	-22	4
P2	.1			.5			.7			.9		

---

[9] A string of three asterisks (='***')indicates that because Y is equal to the maximal demand per period no shortages were observed for that specific parameter set. Therefore no deviations could be computed.

5. Table 4: Y About 30% Above Mean Demand

P1				
.9	4 7 6 -68 -26 -84	20 2 8 -52 -39 -85	0 0 9 *** *** ***	0 0 9 *** *** ***
.7	13 3 5 -53 -13 -63	52 1 7 -34 -26 -46	40 2 8 -40 -40 -48	0 0 9 *** *** ***
.5	4 5 5 -47 -12 -44	18 2 7 -28 -28 -30	10 3 8 -32 -39 -20	0 0 9 *** *** ***
.1	6 4 3 -8 -8 -14	20 2 5 1 -13 -19	10 3 6 -4 -24 -2	2 7 7 0 -29 8
P2	.1	.5	.7	.9

100,000 replications [10]

6. Table 5: All Parameters but Unit Shortage Cost(2) Identical to Table 4, Unit Shortage Cost(2) $ 200 Instead of $ 1,000

P1				
.9	1 4 6 -31 -12 -38	4 2 8 -29 -21 -46	0 0 9 *** *** ***	0 0 9 *** *** ***
.7	4 2 5 -21 -6 -25	12 1 7 -16 -13 -23	8 2 8 -22 -22 -27	0 0 9 *** *** ***
.5	1 4 5 -18 -4 -17	4 2 7 -13 -13 -15	2 3 8 -17 -21 -10	0 0 9 *** *** ***
.1	1 4 3 -3 -3 -6	4 2 5 2 -6 -9	2 3 6 -2 -13 -1	0.4 7 7 3 -13 7
P2	.1	.5	.7	.9

100,000 replications

---

[10] The number of replications had to be increased from 10,000 to 50,000 and finally to 100,000 in order to get the same precision level for the simulation result for Tables 1-5.

7. Analysis of the Simulation Results:

The following summary contains the average absolute deviation (in percent) of the approximation formulas from the simulation results:

	level of Y	monovar. formula	hypergeom. formula	CDF formula
Table 1:	0.85 mean	7.4	5.3	13.8
Table 2:	mean	15.6	11.4	19.5
Table 3:	1.15 mean	23.2	18.5	24.5
Table 4:	1.30 mean	30.5	24.7	38.5
Table 5: with reduced unit shortage cost variability	1.30 mean	14.8	12.2	19.0

In this setting the hypergeometric formula shows slightly more accurate results than the monovariable formula. The CDF formula shows significantly less accurate results than the two other formulas.

Tables 1-4 reveal that for all three approximations the accuracy of the approximations decreases as Y increases.

The accuracies of the approximations are highly dependent on the ratio of the two unit shortage costs of the two demand sources. If both unit shortage costs were the same, the problem is reduced to approximating the expected number of shortages per period. The higher the variability of the unit shortage costs, the less accurate is the approximation of the expected shortage costs per period. To illustrate this property, the variability of the unit shortage costs was set lower in Table 5 than in Tables 1-4. As can be seen when Table 5 is compared to Table 4, the change of unit shortage cost(2) from $ 1,000 to $ 200 reduces the inaccuracies of the approximations to half of their former size.

Although Table 1 represents simulation results with a high variability of the two unit shortage costs, the average absolute deviation for the sixteen runs is 7.4 percent with a maximal absolute deviation of 16 percent for the monovariable approximation. For the hypergeometric formula the average absolute deviation is 5.3 percent with a maximum absolute deviation of 13 percent. These results suggest that the monovariable and the hypergeometric formulas provide an acceptable accuracy when Y is less than the mean of the sum of both demands. Therefore, further analyses will be restricted to settings where Y is equal or greater than the mean of the sum of demands.

C. SECOND SIMULATION SETTING

In this setting the number of repair cycles per demand source were equal. To analyze the effect of smaller increases of Y the three levels are now: Y= mean, Y= 1.10 mean (i.e. only 10 percent increase), and Y= 1.25 mean. The values of P1 and P2 were taken to be 0.1, 0.3, 0.5 and 0.9.

1. Parameter Setting for Tables 6-8:

Number of time units	10
Number of repair cycles(1)	10
Number of repair cycles(2)	10
Unit shortage cost(1)	$ 1000.-
Unit shortage cost(2)	$ 100.-
Initial seed for all runs:	40150

2. Table 6: Y Equal to Mean Demand

10,000 repl's

P1				
.9	434 2 10 5 -3 9	491 2 12 6 -4 21	475 1 14 3 -5 26	285 1 18 -1 -1 -1
.5	590 2 6 4 0 8	542 2 8 3 0 10	490 2 10 -1 -1 -1	328 2 14 -7 4 -41
.3	498 2 4 5 2 13	434 2 6 2 2 2	376 2 8 -2 3 -11	243 2 12 -10 9 -41
.1	278 2 2 2 2 2	223 2 4 -2 5 -18	192 2 6 -6 5 -21	117 2 10 -19 11 -32
P2	.1	.3	.5	.9

3. Table 7: Y About 10 % Above Mean Demand[11]

20,000 repl's

P1				
.9	129 2 11 11 -2 16	205 2 13 9 -4 28	209 2 15 4 -6 33	66 2 19 1 1 1
.5	272 2 7 4 -1 4	283 2 9 3 -1 8	266 2 11 -3 -3 -3	150 2 15 -10 4 -51
.3	231 2 5 3 -1 3	223 2 7 2 2 2	206 2 9 -6 0 -13	113 2 13 -17 7 -51
.1	103 2 3 2 2 2	104 2 5 -5 4 -5	96 2 7 -13 2 -12	41 3 11 -27 14 -42
P2	.1	.3	.5	.9

4. Table 8: Y About 25% Above Mean Demand

50,000 repl's

P1				
.9	5 6 13 5 -16 15	17 3 15 17 -4 47	3 7 18 13 -6 64	0 0 20 *** *** ***
.5	105 2 8 5 -3 -4	128 1 10 4 -2 8	47 2 13 0 0 0	2 8 18 -20 2 -78
.3	89 2 6 2 -3 -6	102 2 8 0 0 0	94 2 10 -7 1 -12	11 4 15 -26 6 -70
.1	32 3 4 -2 -2 -2	41 2 6 -7 4 11	38 2 8 -15 5 8	2 8 13 -43 12 -68
P2	.1	.3	.5	.9

---

[11] The Y's of Table 7 are at least one unit larger than in Table 6; the Y's of Table 8 are at least one unit larger than in Table 7.

5. Analysis of the Simulation Results:

The average absolute deviations (in percent) of the approximation formulas from the simulation results are:

	level of y	monovar. formula	hypergeom. formula	CDF formula
Table 6:	Y = mean	4.9	3.6	16.0
Table 7:	Y = 1.10 mean	7.5	3.4	17.1
Table 8:	Y = 1.25 mean	11.1	4.4	26.2

Tables 6-8 indicate again the relatively strong dependence of the accuracy of the approximations on the size of Y relative to the mean. Although the variability of the unit shortage costs is high, the monovariable and the hypergeometric formulas seem to provide acceptable accuracy even for the case where Y = 1.25 mean. Within this very special setting all three approximations indicate that the accuracy decreases slightly as the difference between P1 and P2 increases. However, Tables 1-8 do not indicate in all cases that the precision of the approximations increase as the p values decrease, as might be expected from the fact that the Poisson distribution is the limiting distribution for the binomial distribution when n becomes large and p becomes small.

D. THIRD SIMULATION SETTING

This setting is a variation on the first setting. In order to get a more general survey about the performances of the approximation formulas the initial seeds and all parameters but the unit shortage costs are different for each run. The unit shortage costs are restricted to $ 100 and $ 1,000. One row refers to one simulation run.

1. Table 9: Runs with Widely Varying Parameters

N1	N2	P1	P2	Pi1	Pi2	MU	Y	SEED	DEVMON	DEVHYP	DEVCDF[12]
1	3	0.8	0.6	1000	100	2.6	3	259	283	230	506
3	4	0.2	0.7	100	1000	3.4	4	999	0	-12	-1
5	3	0.6	0.4	100	1000	4.2	5	255	14	27	22
9	4	0.2	0.8	100	1000	5.0	8	100	236	136	377
3	7	0.6	0.1	1000	100	2.5	3	777	32	10	36
6	5	0.4	0.9	100	1000	6.9	7	654	30	14	59
7	2	0.8	0.1	1000	100	5.8	7	379	-2	-12	-8
3	4	0.6	0.4	1000	100	3.4	5	111	84	62	111
6	1	0.7	0.5	100	1000	4.7	5	7	98	118	151
2	7	0.8	0.6	1000	100	5.8	6	825	245	216	355
3	2	0.5	0.7	100	1000	2.9	3	900	32	20	38
7	4	0.2	0.9	1000	100	5.0	5	693	-24	3	-50
5	9	0.6	0.3	1000	100	5.7	7	387	17	4	33
3	11	0.1	0.7	100	1000	8.0	9	951	-1	-4	-1
6	7	0.4	0.6	1000	100	6.6	8	291	22	34	15

Each run: 20,000 replications

2. Analysis of the Simulation Results:

The average absolute deviations (in percent) are: DEVMON = 74.7, DEVHYP = 60.1, and DEVCDF = 117.5. These error deviations are relatively large. However, a reduction in the variability of the unit shortage cost to $ 100 and $ 200 would reduce the average absolute deviations by about one-half demonstrated in Tables 4 and 5.

---

[12] Abbreviations in Table 9:
N1 - number of repair cycles(1)
N2 - number of repair cycles(2)
P1 - p parameter(1)
P2 - p parameter(2)
Pi1 - unit shortage cost(1)
Pi2 - unit shortage cost(2)
MU - expected sum of demand per period
Y - purchased quantity
seed - initial seed used for result in current row
DEVMON, DEVHYP, and DEVCDF represent the deviations of the approximations from the simulation result in percent.

E. FOURTH SIMULATION SETTING

This simulation setting is similar to both the second and third setting. As in the second setting, the number of repair cycles is the same for both demand sources. As in the third setting, the parameters have row by row comparable values. A comparison between corresponding rows in Table 9 and Table 10 should indicate whether the performances of the approximation formulas improve if the lower number of repair cycles is increased such that both demand sources have the same number of repair cycles. The ratio Y/MU is about the same as in Table 9.

1. Table 10: Identical Parameters as Table 9; But N1=N2

N1	N2	P1	P2	Pi1	Pi2	MU	Y	SEED	DEVMON	DEVHYP	DEVCDF[13]
3	3	0.8	0.6	1000	100	4.2	5	175	12	1	34
4	4	0.2	0.7	100	1000	3.6	4	231	11	-3	14
5	5	0.6	0.4	100	1000	5.0	6	725	-5	5	-15
9	9	0.2	0.8	100	1000	9	12	798	13	-7	28
7	7	0.6	0.1	1000	100	4.9	6	183	7	-1	4
6	6	0.4	0.9	100	1000	7.8	8	436	10	-3	31
7	7	0.8	0.1	1000	100	6.3	7	123	6	-5	6
4	4	0.6	0.4	1000	100	4.0	5	123	9	0	17
6	6	0.7	0.5	100	1000	7.2	8	190	-4	4	-20
7	7	0.8	0.6	1000	100	9.8	10	850	3	-2	21
3	3	0.5	0.7	100	1000	3.6	4	136	7	-2	17
7	7	0.2	0.9	1000	100	7.7	8	212	-17	15	-41
9	9	0.6	0.3	1000	100	8.1	10	187	1	-7	7
11	11	0.1	0.7	100	1000	8.8	10	449	7	-1	5
7	7	0.4	0.6	1000	100	7.0	8	725	-1	7	-11

Each run: 20,000 replications

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[13] The abbreviations are the same as for Table 9. (see preceding footnote)

2. Analysis of the Simulation Results:

The average absolute deviations (in percent) are: DEVMON = 7.5, DEVHYP = 4.2, and DEVCDF = 18.1. The simulation results establish that the monovariable and the hypergeometric approximation formulas provide better accuracy when the number of repair cycles is the same for both demand sources.

F. FIFTH SIMULATION SETTING

The increase of the lower number of repair cycles of Table 9 such that both demand sources had an equal number of repair cycles resulted in a dramatic improvement in the accuracy of all three approximation formulas. This improvement could be caused by two facts. The first is that the approximations improve when there is an identical number of repair cycles, as established in the second and fourth simulation settings. The second is that an increase of the parameter n is one condition for the convergence of the binomial distribution to the Poisson distribution. Therefore the improvement could be caused simply by the increased number of repair cycles.

Table 11 displays results of simulation runs with the number of repair cycles of one demand source of Table 10 further increased. The ratio Y/MU is about the same as in Table 10. Comparison of Tables 9,10, and 11 provides an indication of both the impact of increasing the n values and having them equal in value.

1. Table 11: Continuation of Table 10: But N1 or N2 Increased

N1	N2	P1	P2	Pi1	Pi2	MU	Y	SEED	DEVMON	DEVHYP	DEVCDF[14]
3	5	0.8	0.6	1000	100	5.4	6	212	43 W	31 W	77 W
5	4	0.2	0.7	100	1000	3.6	5	542	-2 I	-13 W	-3 I
5	7	0.6	0.4	100	1000	5.8	7	239	-13 W	-7 W	-23 W
9	10	0.2	0.8	100	1000	9.8	13	822	-11 I	-26 W	-2 I
9	7	0.6	0.1	1000	100	6.1	7	17	3 I	-3 W	2 I
6	7	0.4	0.9	100	1000	8.7	9	489	-7 I	-17 W	9 I
8	7	0.8	0.1	1000	100	7.1	8	899	0 I	-10 W	-2 I
4	6	0.6	0.4	1000	100	4.8	6	117	26 W	15 W	41 W
6	8	0.7	0.5	100	1000	8.2	9	299	-15 W	-10 W	-26 W
7	9	0.8	0.6	1000	100	11	11	168	17 W	11 W	33 W
5	3	0.5	0.7	100	1000	4.6	5	256	20 W	11 W	32 W
7	8	0.2	0.9	1000	100	8.6	9	437	8 I	51 W	-21 I
11	9	0.6	0.3	1000	100	9.3	11	219	-1 E	-7 E	1 I
22	11	0.1	0.7	100	1000	9.9	11	612	15 W	5 W	22 W
9	7	0.4	0.6	1000	100	7.8	9	439	-13 W	-7 E	-23 W

Each run: 20,000 replications

2. Analysis of the Simulation Results:

The average absolute deviations (in percent) are: DEVMON = 12.9, DEVHYP = 14.9, and DEVCDF = 21.1.

In general, the accuracy of the approximations decreased. However, when compared to Table 10, the monovariable formula improved its performance in 6 out of 15 cases, and the CDF formula in 7 out of 15 cases, whereas the hypergeometric formula lost accuracy in 14 out of 15 cases.

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[14] The additional letters in columns DEVMON, DEVHYP, and DEVCDF indicate:
I - improved
E - same level
W - became worse

It can be concluded that an increase of the number repair cycles has a larger improving influence on the accuracy of the monovariable and the CDF formulas than it has on the accuracy of the hypergeometric formula. The accuracy of the hypergeometric formula is more sensitive to the ratio of N1 to N2 than the two others; if performs best when N1 = N2.

The impact of increasing n can quickly be seen when the average absolute deviations of Table 11 are compared to Table 9.

	DEVMON	DEVHYP	DEVCDF
Table 9:	74.7	60.1	117.5
Table 11:	12.9	14.9	21.1

Table 9 and 11 display results for very similar parameter values. In corresponding rows all parameters values are identical except N1,N2 and Y. However, in each row of Table 11 there are increased but unequal numbers of repair cycles, the Y's of Table 11 are larger than in Table 9 in order to keep the ratio Y/MU at about the same level for corresponding rows.

G. SIXTH SIMULATION SETTING

Up to this point the timing of demands has been assumed to be at the start of the repair cycle. However, this may not always be the case at the NARF; the timing is dependent on the length of time for teardown and reassembly of a repairable. It is quite possible that demand for an item could occur at the end of a cycle when the reapairable is in its final stage of reassembly. In addition, in Chapter II we illustrated the impact of the timing of the demands during the repair cycle on the shortage costs with a simple example.

Another assumption has been that, in the event of two customers simultaneously demanding the last unit in inventory , the supply department would toss a coin to see who gets the item. However, it is not unreasonable that the supply department might favor the customer with the higher expected demand per period.

In this simulation setting demands are assumed to occur at the end of the repair cycle and, in the case of a tie for the last unit, the probability that a customer will get it is the ratio of his mean demand for the period to the total mean demand from all customers. The simulations were reruns of Table 9 with "Stage 1" involving the timing change and "Stage 2" involving both the timing change and the new priority rule.

1. Analysis of the Simulation Results:

The average absolute deviations (in percent) are:

	DEVMON	DEVHYP	DEVCDF
In Table 9:	74.7	60.1	117.5
For Stage 1:	17.8	18.9	21.5
For Stage 2:	10.1	17.1	17.7

These results establish with very high significance the decisive influence of the timing of the demands on the accuracies of all three appproximation formulas. They also suggest that, if the supply department would consider a priority scheme based on expected demands rather than tossing a coin for the last unit, the shortage cost approximations could be used with more confidence.

Interestingly, the occurrence of demand within the repair cycle does not matter, when the number of repair cycles is identical because the cycles are then of identical length too. In the case of an equal number of repair cycles, the order in which demands occur will not change as

2. Table 12: Rerun of Table 9 with Changed "Real World"

Accuracies achieved in Table 9			Stage 1, shifted occurrence of demand			Stage 2, additionally changed priority rule		
D-MON	D-HYP	D-CDF	D-MON	D-HYP	D-CDF	D-MON	D-HYP	D-CDF
283	230	506	-31	-40	10	2	-13	61
0	-12	-1	16	2	15	5	8	4
14	27	22	-21	-12	-15	-9	2	-3
236	136	377	25	-13	77	15	-19	64
32	10	36	4	-13	7	1	-15	5
30	14	59	10	-4	34	5	-9	28
-2	-12	-8	33	20	26	5	-6	-1
84	62	111	-1	-13	14	-4	-15	11
98	118	151	-61	-57	-50	-45	-40	-31
245	216	355	-37	-42	-17	-30	-35	-7
32	20	38	-9	-17	-5	-8	-16	-4
-24	3	-50	-1	34	-35	7	45	-30
17	4	33	-6	-16	8	-6	-16	7
-1	-4	-1	11	8	10	7	4	7
22	34	15	-2	7	-9	3	13	-3

Each run: 20,000 replications

long as demands occur at the same point in time within the repair cycles for both demand sources. This fact explains the relatively good performance of the approximation formulas in simulation settings 2 and 4. In the case of an unequal number of repair cycles the order will generally change.

H. SEVENTH SIMULATION SETTING

As the number of repair cycles increases, the influence of the timing of demands within a repair cycle on the accuracy of the approximations will decrease.

Simulation settings 1-6 provided strong evidence that the CDF approximation formula is clearly the worst. It does not provide sufficient accuracy and will be excluded from further simulations and analysis.

The following restrictions for the parameter values should ensure an acceptable accuracy for the monovariable and the hypergeometric approximation formulas when they are applied to the binomial NARF model. The restrictions are derived from simulation settings 1-6:

1. The number of repair cycles for each demand source is greater than or equal to 10.
2. The mean of the sum of both demands per period plus 1.50 times its standard deviation is an upper bound for Y. [15]
3. The ratio of the larger unit shortage cost to the smaller is at most three.

Table 13 represents results of simulations which were run with parameter values within the above restrictions.

1. Analysis of the Simulation Results:

The average absolute deviations (in percent) are: DEVMON = 6.3 and DEVHYP = 5.8. The maximal absolute deviation is 14 percent for the monovariable formula and 16 percent for the hypergeometric formula.

Because the fifteen simulation runs are independent observations for DEVMON and DEVHYP, nonparametric statistical analysis can be performed. Conover [Ref. 8 p.118] provides a nonparametric method for computing tolerance limits. His method applied to Table 13 establishes that, with a confidence level of 95 percent, at least 81 percent

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[15] If Y is equal to the mean of the sum of demands per period plus 1.5 of its standard deviations, shortages will occur in about one out of 15 periods. This quantification is a rough estimate derived from the standardized normal distribution, i.e. P(Z>1.5)=0.067.

2. Table 13: Runs with Restricted Varying Parameters [16]

N1	N2	P1	P2	MU	STD	Y	Pi1	Pi2	Seed	SHCOST	DEVMON	DEVHYP
10	15	0.2	0.8	14.0	2.0	16	100	200	909	28	-1	-5
12	18	0.4	0.6	15.6	2.7	19	300	100	109	19	5	9
15	12	0.7	0.6	17.7	2.5	20	100	300	891	33	14	16
10	20	0.3	0.5	13.0	2.7	17	300	100	510	11	6	11
11	22	0.1	0.9	20.9	1.7	23	100	300	175	21	3	-2
21	14	0.3	0.2	9.1	2.6	12	300	100	175	43	0	-1
12	16	0.8	0.1	11.2	1.8	13	100	250	311	20	-12	0
20	40	0.9	0.9	54.0	2.3	56	100	300	113	55	-13	-13
10	15	0.2	0.2	5.0	2.0	8	100	300	975	16	-2	-2
20	15	0.2	0.6	13.0	2.6	16	300	100	305	30	-12	-5
12	18	0.9	0.5	19.8	2.4	23	100	300	129	18	-11	-4
13	39	0.4	0.7	32.5	3.4	37	100	300	610	36	-5	-7
11	33	.05	0.1	3.9	1.9	7	300	100	711	6	5	7
18	12	0.2	0.3	7.2	2.3	8	300	100	111	118	-2	0
10	40	0.3	0.5	23.0	3.5	25	100	300	637	173	-4	-5

Each run: 20,000 replications

of the absolute deviations of the monovariable and hypergeometric formulas will not exceed the maximal absolute deviations of Table 13.

I. SUMMARY AND CONCLUSIONS

This chapter has shown that the monovariable and the hypergeometric formulas provide significantly more accurate results than the CDF formula does. The following additional remarks refer to the monovariable and the hypergeometric formulas.

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[16] STD denotes the standard deviation for the sum of both demands per period. SHCOST denotes the average value for the shortage costs per period as observed during simulation.

All simulations show a clearly recognizable trend that the accuracy decreases as Y increases with levels of Y which correspond to most "real world" situations (i.e. mean -15%, mean +30%). This suggests that the approximations are not good in the upper tails of the demand distributions.

The simulations suggested that the approximations provide their most accurate results when there is an identical number of repair cycles. Even in the case of a high variability of the unit shortage costs (i.e. $ 1,000 and $ 100), the average absolute deviation from the simulation results (simulation settings 2 and 4) is about 10 % for the monovariable formula and 5 % for the hypergeometric formula.

The variability of the unit shortage costs is an important factor for the accuracy of the approximation formulas . Reducing the variability of the unit shortage costs from $ 1,000 and $ 100 to $ 200 and $ 100 almost doubles the accuracy to be expected from both approximations, as was demonstrated in simulation setting 1.

Simulation settings 4 and 6 suggest that the approximations perform significantly better when the demands are assumed to occur at the end of the repair cycle. The influence of the timing of the demands on the accuracy of the approximation formulas decreases as the number of repair cycles increases.

In settings with different numbers of repair cycles per period the monovariable and the hypergeometric formulas provide about the same level of accuracy. By imposing the following restrictions for most practical applications an acceptable level of accuracy can be expected:

1. The number of repair cycles for each demand source is greater than or equal to 10.

2. The mean of the sum of both demands per period plus 1.50 times its standard deviation is an upper bound for Y.
3. The ratio of the larger unit shortage cost to the smaller is at most three.

More specifically, when observing these restrictions it is to be expected that, with a confidence level of 95 percent, at least 81 percent of the actual absolute deviations of the monovariable formula will not exceed 14 percent, and, in the case of the hypergeometric formula, they will not exceed 16 percent.

VI. MORE THAN TWO BINOMIAL DEMAND SOURCES

A. PRELIMINARY REMARKS

This chapter analyzes the situation when there are more than two demand sources and presents results from the associated simulation runs. In particular, it considers the approximation of the distribution of the sum of demands per period by the normal distribution.

The computational effort required to use the hypergeometric formula increases very rapidly with an increasing number of demand sources. Therefore only the monovariable formula will be evaluated for the case of more than two individual demand sources.

B. METHODS FOR ESTIMATING THE EXPECTED SHORTAGE COSTS PER PERIOD

Two basic ways for estimating the expected shortage costs per period in the case of "many" binomial distributed demand sources are analyzed in this section. The first is to simulate the complete period for a sufficient number of replications to provide the desired precision. If the desired precision has been prespecified, the number of necessary replications can be determined by the program during its execution. This simulating approach gives an unbiased estimate for the expected shortage costs per period and, additionally, allows for varying the time at which the demands are assumed to occur within the repair cycle.

The second way to estimate the expected shortage costs per period is to apply the monovariable approximation formula. However, before applying the monovariable formula the distribution for the aggregate demand has to be computed.

In the case of "many" binomially distributed demand sources with different p parameters there are essentially four approaches for computing the distribution of aggregate demand:

1. The first approach is to estimate the distribution of the sum of demands per period by simulation. The demand occurrences during the period are simulated and added up to get one observation for the sum of demands per period. This is repeated for a specified number of replications. The observed empirical distribution for the sum of demands during simulation can be taken as an estimate for the distribution of the sum of demands per period.
2. The second approach is to compute the exact distribution of the sum of demands. However, the number of computations for computing the exact distribution grows as a factorial expression whereas the number of computations performed during simulation only grows approximately linearly along with the number of demand sources. For instance, if there are five demand distributions with <6,7,8,9,10> repair cycles per period, then a five-fold convolution is required which incorporates 7x8x9x10x11 = 55,440 iterations.
3. There are two analytical approximating methods which can reduce the huge number of computations involved in finding the distribution of the sum of the demands per period. One approach is to approximate the individual binomial distributions by Poisson distributions.
4. The second analytical method is to approximate the individual distributions by normal distributions.

C. APPROXIMATION OF THE SUM OF DEMANDS BY THE NORMAL DISTRIBUTION

An advantage of the monovariable approximation formula is that it does not require the consideration of all possible permutations. It requires only the distribution of the sum of demands per period. Therefore it can be applied when the individual binomial distributions are approximated by normal distributions. The sum of normal random variables is again normally distributed. Its mean and variance are the sum of the individual means and variances [Ref. 9 p.405].

The Poisson distribution could also be used as an approximation since the sum of Poisson distributed random variables is a Poisson random variable whose mean is the sum of the individual means. [Ref. 9 p.405].

The advantage of the normal distribution over the Poisson is that it has two parameters. One parameter determines the level (=mean) and the other one the spread (=variance). The effects of the two parameters are mutually independent and permit the modelling of a wider variety of demand behavior. Therefore the normal distribution is often preferred over the Poisson distribution when one attempts to fit a theoretical distribution to an empirical distribution.

A continuity correction was used to reduce the error of approximating a discrete distribution with a normal distribution. If, for instance, the probability associated with the integer I is to be computed, the normal approximation with continuity correction uses the area under the normal density from (I - 0.5) to (I + 0.5) to approximate the probability.

A further reason for using the normal distribution is a consequence of the Central Limit Theorem. From the Central Limit Theorem it follows that as the number of demand sources increases, the aggregate distribution of demands per period will be increasingly better approximated by the normal distribution.

D. RUNNING THE SIMULATION

In case of an arbitrary number of demand distributions the monovariable approximation formula uses an approximating distribution to evaluate the expected number of shortages per period. Therefore, deviations between the result of simulation and the monovariable approximation formula when aggregate demand is assumed to be normally distributed might be caused by both the process of applying the approximation formula and this use of the normal distribution.

The output tableau of the simulation program is basically an extension of the output tableau described in Chapter 4, section D of this thesis. To enable an efficient output analysis, the program provides more key statistics than the former program for two binomial demand distributions.

```
-----------------------------------------------------------------
NOFDMD= 2[1] #OF TIME UNITS= 30[2] REPLI=50000  I-SEED=      909

 #OF REP-     PROBA-                        UNIT-SHORT.      FIT[3]
 CYCLES       BILITY         MEAN           COST           N*P   N*Q

  1: 10       1: 0.20        1:    2.00     1:   100.00     2.0   8.0
  2: 15       2: 0.80        2:   12.00     2: . 200.00    12.0   3.0

TOTAL MEAN= 14.00   TOTAL S-DEV= 2.00[4]  Y= 16   MAX-DMD=  25

      SIMULATION: AVE-COST=    28.36[5] #OF SHOR'S=   0.1525[6]
SHSIMU= 10.30%      SD-COST=    0.42[7] SD-#OF SH'S=  0.0023[7]

NORMAL-APPROXIM.: AVE-COST=    30.01  #OF SHOR'S=   0.1616

SUMMARY STATISTICS:
SD/AVE=     1.47%[8]  PISIMA=  185.97$[9]   PINORA=  135.71$[10]
DEVMON=     5.83%[11] DEVNSH=    5.97%[12]  DEVPIA=   -0.14%[13]
-----------------------------------------------------------------
```

1. Number of demand sources;
2. Number of time units per period;
3. The last two columns indicate how well the individual binomial distribution might be approximated by an normal distribution. In general, n*p and n*(1-p) should both have at least a value of five to ensure a sufficient fit for the individual distribution. However, applying the monovariable formula incorporates only the distribution of aggregate demands. With an increasing number of demand sources it is to be expected that according to the Central Limit Theorem the approximation of the aggregate demand distribution will be by far better than indicated by the N*P and N*(1-P) values for the individual distributions.
4. Standard deviation of the sum of demands;
5. Estimated expected shortage costs per period;
6. Average number of shortages per period as they were observed during simulation.
7. The observed standard deviations for the estimates give a numerical hint about the precision of the estimates.
8. The observed standard deviation of the sample mean divided by the sample mean of the shortage costs per period.
9. Average unit shortage costs derived from the simulation, i.e. accumulated observed shortage costs divided by the observed number of shortages.
10. Average unit shortage costs weighted by the means of the individual demand distributions corresponding to the way the monovariable approximation formula uses these weights.
11. Deviation of the result of the monovariable approximation formula from the estimate for the expected shortage costs per period derived from simulation.

12. The deviation of the average number of shortages used by the monovariable approximation formula from the average number of shortages as observed during simulation.
13. The deviation of the mean weighted average unit shortage cost computed when applying the monovariable approximation formula from the observed average unit shortage cost. DEVNSH and DEVPIA provide a rough means for distinguishing whether the deviation between simulation results and monovariable approximation is caused by the approximation formula or by the approximation of the distribution of the aggregate demands per period by the normal distribution.

E. SIMULATION RESULTS

This section presents conclusions derived from simulation runs performed in order to evaluate the performance of the monovariable approximation formula when the total demand distribution is approximated by the normal. For all simulations presented in this section the range of possible parameter values is within the limits of the three restrictions listed at the end of Chapter V.

Table 14 is a direct continuation of Table 13. The only difference is that the distribution of the sum of demands is now approximated by a normal distribution.

The average absolute deviations (in percent) are: DEVMON = 11.1, DEVNSH = 10.4, and DEVPIA = 5.8. The 'DEVPIA' column of Table 14 is almost identical to the 'DEVMON' column of Table 13. The 'DEVNSH' column of Table 14 measures the additional inaccuracy incurred by applying the normal approximation instead of using the exact distribution for the sum of demands as was done in Table 13.

1. Table 14: Runs with Identical Parameters as Table 13 [17]

N1	N2	P1	P2	MU	STD	Y	Pi1	Pi2	Seed	DEVMON	DEVNSH	DEVPIA
10	15	0.2	0.8	14.0	2.0	16	100	200	909	5.8	6.0	-0.1
12	18	0.4	0.6	15.6	2.7	19	300	100	109	9.4	5.4	3.8
15	12	0.7	0.6	17.7	2.5	20	100	300	891	22.9	10.8	11.0
10	20	0.3	0.5	13.0	2.7	17	300	100	510	5.1	-1.7	6.9
11	22	0.1	0.9	20.9	1.7	23	100	300	175	22.9	24.1	-1.0
21	14	0.3	0.2	9.1	2.6	12	300	100	175	-8.1	-7.7	-0.4
12	16	0.8	0.1	11.2	1.8	13	100	250	311	-8.4	2.7	-10.8
20	40	0.9	0.9	54.0	2.3	56	100	300	113	-2.1	11.2	-12.0
10	15	0.2	0.2	5.0	2.0	8	100	300	975	-23.1	-20.1	-3.8
20	15	0.2	0.6	13.0	2.6	16	300	100	305	-14.2	-6.1	-8.6
12	18	0.9	0.5	19.8	2.4	23	100	300	129	-1.0	14.4	-13.5
13	39	0.4	0.7	32.5	3.4	37	100	300	610	3.8	8.8	-4.6
11	33	.05	0.1	3.9	1.9	7	300	100	711	-33.4	-36.1	4.2
18	12	0.2	0.3	7.2	2.3	8	300	100	111	-2.8	0.2	-3.1
10	40	0.3	0.5	23.0	3.5	25	100	300	637	-3.9	-1.1	-2.8

Each run: 20,000 replications

If in one row of Table 14 the actual values of DEVNSH and DEVPIA are either both negative or both positive then the actual overall deviation(DEVMON) represents an accumulation of DEVNSH and DEVPIA. Otherwise, when the signs of the actual values for DEVNSH and DEVPIA are different then the aggregate affect of DEVNSH and DEVPIA on the value of DEVMON results in a partial compensation of their individual affects. In Table 14 the actual values for DEVNSH and DEVPIA have different signs in nine out of fifteen rows.

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[17] DEVNSH = deviation in number of short's(in percent) ;
DEVPIA = deviation in average unit shortage cost (in percent).

The average absolute deviations of DEVNSH and DEVPIA indicate that about two thirds of the overall average absolute deviation is caused by using the normal distribution as an approximation for the distribution of the sum of demands and the rest from applying the monovariable approximation formula. It can be concluded that in the case of two demand sources it is worthwhile to compute the exact distribution of the sum of demands if the monovariable approximation formula is going to be applied.

Simulation results for three demand sources are displayed in Appendix C. The mean of the distribution of the sum of demands is equal to 21.6 demands per period and its standard deviation is equal to 3.75. These tables show that the accuracy of the overall approximation decreases with increasing Y. The following results (in percentages) were obtained:

	DEVMON	DEVNSH	DEVPIA
Y = 20	-1.3	0.3	-1.6
Y = 23	-3.4	-1.2	-2.2
Y = 26	-7.4	-4.7	-3.1

These results indicate again that the accuracy decreases as Y increases.

Appendix D contains results of fifteen simulations for three demand sources. The fifteen cases of Table 14 were expanded by one additional demand source in order to provide a basis for comparing the results of Appendix D with Table 14. For each tableau of Appendix D the ratio Y/MU is about the same as in corresponding rows of Table 14.

The average absolute deviations (in percent) are: DEVMON = 5.8, DEVNSH = 4.2, DEVPIA = 3.8. For Table 14 they were: 11.1, 10.4, and 5.8, respectively. The expansion to three demands sources shows already a strong numerical influence from the Central Limit Theorem. The average absolute deviation for the number of shortages dropped from 10.4% to 4.2%. In six out of fifteen cases the actual values of DEVNSH and DEVPIA have different signs which reduces the overall deviation for those cases.

Conover's method [Ref. 8 p.118] for computing tolerance limits applied to Appendix D establishes that, with a confidence level of 95 percent, at least 81 percent of the absolute deviations(DEVMON) will not exceed 11.9 percent if the three restrictions for the ranges of parameter values are applied. This accuracy should be acceptable for most applications.

Appendix E contains results of simulation runs for five demand sources with one parameter set and three levels of Y, i.e. mean, mean plus one standard deviation, and mean plus 1.5 standard deviations. The deviations (in percent) are:

	DEVMON	DEVNSH	DEVPIA
Y = 42	0.9	0.0	0.9
Y = 46	0.5	-1.7	1.2
Y = 48	-0.2	-2.3	2.2

These results indicate that an acceptable level of accuracy can be expected in the cases of five and more demand sources.

F. SUMMARY AND CONCLUSIONS

This chapter has shown that in case of two demand sources it is worthwhile to compute the exact distribution of the aggregate demands. In the case of three demand sources, the benefit gained from computing the exact distribution for the sum of demands is significantly lower. However, if the numbers of repair cycles are small - which means a relatively small computational effort is required in computing the exact distribution of the sum of demands - it is recommended that the exact distribution for the sum of demands be computed instead of using the normal approximation. Otherwise for the cases of three or more demand sources the normal approximation should be applied.

All simulation results presented in this chapter indicate that the sign of the actual deviations caused from approximating the distribution of the aggregate demands (DEVNSH) is independent from the sign of the deviation resulting from applying the monovariable approximation formula (DEVPIA). If for a given parameter setting the signs of DEVNSH and DEVPIA are different then the overall deviation (DEVMON) represents a partial compensation of the aggregated affects of DEVNSH and DEVPIA; otherwise, the overall deviation is about equal to the sum of DEVNSH and DEVPIA. Therefore, the impact of the normal approximation on the variability of the accuracy will be stronger than its impact on the expected level of accuracy.

VII. SUMMARY AND SUGGESTIONS FOR FURTHER ANALYSIS

A. SUMMARY

Chapter II proved that the monovariable approximation formula for the expected shortage costs per period provides the exact value for the expected shortage costs per period in the case of any number of Poisson distributed individual demands. Because the monovariable approximation does not provide the exact value for the expected shortage costs per period in the case of binomially distributed individual demands as they are modelled by the NARF model, two further approximation formulas, the hypergeometric formula and the CDF formula, were introduced in Chapter III.

Chapter IV described the simulation methodology which was used in Chapter V to evaluate the performances of the three approximation formulas in the case of two binomially distributed demand sources. Chapter V showed that the monovariable and the hypergeometric formulas provide about the same level of accuracy. Both formulas provide significantly more accurate results than the CDF formula. They are therefore the ones that should be used in applications.

Chapter V showed that three factors are significant in determining the accuracy of the three approximation formulas:

1. The occurrence of demand at the end of the repair cycle improves the accuracy of the approximations. However, in almost all "real world" situations which are modelled by the NARF model, demand will occur at the beginning of the repair cycle. The approximation error incurred hereby will decrease as the numbers of repair cycles increase.

2. The accuracy of the approximations for the expected shortage costs per period decreases as the available quantity Y increases reflecting the fact that the approximations are not so good in the upper tails of the distributions.
3. If both unit shortage costs were the same, the problem of approximating the expected shortage costs per period is reduced to approximating the expected number of shortages per period. The higher the variability of the unit shortage costs, the less accurate is the approximation for the expected shortage costs per period.

By imposing three restrictions on the range of parameter values, an acceptable expected level of accuracy can be achieved for the monovariable and the hypergeometric formula in the cases of two demand sources. These restrictions are:

1. The number of repair cycles for each demand source is greater than or equal to 10.
2. The mean of the sum of both demands per period plus 1.50 times its standard deviation should be an upper bound for Y. [18]
3. The ratio of the larger unit shortage cost to the smaller is at most three.

Tolerance limits for the accuracy were computed from simulation results in Chapter V. With a confidence level of 95 percent, at least 81 percent of the actual results from the monovariable approximation formula will incur at most an absolute deviation of 14 percent from the exact numerical value of the expected shortage costs per period, and at most 16 percent in case of the hypergeometric formula. As a general tendency, the hypergeometric formula seems to perform slightly better than the monovariable approximation formula, especially in the case when there are equal numbers of repair cycles associated with each demand source.

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[18] If Y is equal to the mean of the sum of demands per period plus 1.5 of its standard deviations shortages will occur in about one out of 15 periods. This quantification is a rough estimate derived from the standardized normal distribution, i.e. $P(Z>1.5)=0.067$.

Chapter VI expanded the setting to more than two binomially distributed demand sources. The problem with the hypergeometric formula is that all possible permutations of demands must be evaluated whereas the monovariable formula requires only the distribution of the aggregate demands per period. Because the number of computations needed to apply the hypergeometric formula increases rapidly as the number of demand sources is increased, only the monovariable formula was evaluated in the cases of more than two demand sources.

Chapter VI showed that in case of two demand sources it is worthwhile to compute the exact distribution of the aggregate demands rather than using the normal distribution to approximate the distribution of aggregate demands. In the case of three demand sources, the benefit gained from computing the exact distribution for the sum of demands was significantly lower due to the increased computational effort required. However, if the numbers of repair cycles are small, that is not much of a problem and it is recommended that in the cases of three or four demand sources that the exact distribution for the sum of demands be computed. Otherwise for the cases of three or more demand sources the normal approximation can successfully be applied if certain parameters are held within a limited range of values.

All simulation results presented in Chapter VI indicated that the signs of the actual deviations caused by approximating the distribution of the aggregate demands were independent from the signs of the deviations resulting from applying the monovariable approximation formula. If, for one specific parameter setting, the signs are different then their effects on the overall deviation get partially compensated. Otherwise, the overall deviation is about equal to the sum of the two single deviations.

B. FACTORS TO BE CONSIDERED FOR PRACTICAL APPLICATIONS

In case of binomially distributed demand sources the following factors should be considered.

The NARF model incorporates mutually independent binomially distributed demands. As was indicated in Chapter II, the Poisson distribution can be used to approximate the binomial distribution. The approximation provides an acceptable level of accuracy if $n \geq 20$ and $p \leq 0.05$; or if $n \geq 100$ and $np \leq 10$ [Ref. 10 p.79]. If the actual demand distributions at the NARF all exhibit such parameter values then the monovariable formula can be used along with the Poisson distribution for the aggregate demands.

As a last resort, simulation of the actual demand behavior may be the only approach for determining the expected shortage costs per period. In particular, if one of the restrictions listed above is violated, simulation should be used to estimate the expected shortage costs.

C. SUGGESTIONS FOR FURTHER ANALYSIS

The approximation for the expected shortage costs appeared to be more accurate when the timing of demand moved to the end of the repair cycle and when the priority discipline was based on expected demand. In the case of the latter result, a minor change in the policy of running the system is suggested.

As the stock gets more and more depleted during the period a rational policy would be to reserve stock for customers with the higher unit shortage costs and expected higher demands. The higher the variability of the customer specific unit shortage costs and the lower the holding cost per stocked item, the greater is the benefit from switching from a "first-come, first-served" rule to a more sophisticated rule which allows for the different economic importance of the different demand sources. Analysis has to show whether the costs for implementing this flexible policy are justified by cost reductions in the sum of economic costs associated with running the inventory system.

In the case of the former result, further study of the influence of timing is warranted. Perhaps a shortage cost approximation can be developed which includes the timing as a parameter.

APPENDIX A

COMPUTER PROGRAM FOR SIMULATING TWO BINOMIAL DEMAND SOURCES

```
$JOB                                 NAME: WILHELM F. NOEGGERATH
C                                          DATE: JULY, 18TH 1984
C
C       BINOMIAL DISTRIBUTION, TWO DEMAND SOURCES
C           - SIMULATION OF BERNOULLI TRIALS PER REPAIR CYCLE
C           - MONOVARIABLE APPROXIMATING FORMULA
C           - HYPERGEOMETRIC ORIENTATED APPROXIMATION FORMULA
C           - CDF ORIENTATED APPROXIMATION FORMULA
C
C   VARIABLES:
C   APPROX RESULT OF MONOVARIABLE APPROXIMATING FORMULA
C   AUX..  AUXILIARY VARIABLES
C   AVECOS AVERAGE COST PER PERIOD
C   BIAELI ELIMINATES BIAS IF TWO DEMANDS IN ONE TIME UNIT
C          AND ONE UNIT OF STOCK LEFT ONLY
C   COSCDF RESULT OF CDF- ORIENTATED APPROXIMATION FORMULA
C   COSCRT COST FOR CURRENT SIMULATED PERIOD
C   COSCUM CUMULATED COST FOR ALL SIMULATED PERIODS
C   COSHYP RESULT OF HYPERGEOM. ORIENTATED APPROXIM. FORMULA
C   COSSQ  CUMULATED PERIOD SQUARED COSTS
C   CTR    COUNTS PERIODS IN WHICH SHORTAGES OCCUR DURING
C          SIMULATION
C   DEVCDF DEVIATION IN PERCENT OF RESULT CDF ORIENTATED
C          APPROXIMATING FORMULA FROM RESULT OF SIMULATION
C   DEVHYP DEV. IN PERCENT RESULT HYPGEO. APPROXIM. FORMULA
C   DEVMON DEV. IN PERCENT RESULT MONOVAR. APPROXIM. FORMULA
C   DMD    SCALAR, CUMULATIVE DEMAND IN ONE PERIOD
C   DMD1   ARRAY OF DEMANDS WITHIN ONE PERIOD CAUSED
C          BY END ITEM #1
C   DMD2   ARRAY OF DEMANDS WITHIN ONE PERIOD CAUSED
C          BY END ITEM #2
C   INTVL1 LENGTH OF REPAIR  CYCLE FOR END ITEM #1
C   INTVL2 LENGTH OF REPAIR CYCLE FOR END ITEM #2
C   ISEED  INITIAL SEED, GETS PRINTED
C   ITMQIT INDICATES FOR WHICH END ITEM IN CURRENT SIMULATED
C          PERIOD LAST DEMAND SATISFIED
C   MU     SUM: MU1 + MU2
C   MU1    MEAN BINOMIAL-DIST. DEMAND CAUSED BY END ITEM #1
C   MU2    MEAN BINOMIAL-DIST. DEMAND CAUSED BY END ITEM #2
C   N      NUMBER OF DISCRETE TIME UNITS PER PERIOD
C    !! ****  N MUST BE A MULTIPLE OF N1 AND OF N2   **** !!
C   N1     NUMBER OF REPAIR CYCLES END ITEM #1 IN ONE PERIOD
C   N2     NUMBER OF REPAIR CYCLES END ITEM #2 IN ONE PERIOD
C   PI1    SHORTAGE COST ITEM #1
C   PI2    SHORTAGE COST ITEM #2
C   PMF    PROB. MASS FCT FOR DMD PER PERIOD CAUSED
C          BY E-ITEMS #1 AND #2
C   PMF1   PROBABILITY MASS FCT FOR DMD PER PERIOD
C          CAUSED BY E-ITEM #1
C   PMF2   PROBABILITY MASS FCT FOR DMD PER PERIOD
C          CAUSED BY E-ITEM #2
C   P1     PROBAB. OF DEMAND CAUSED BY END ITEM #1
C          IN ONE REPAIR CYCLE
C   P2     PROBAB. OF DEMAND CAUSED BY END ITEM #2
C          IN ONE REPAIR CYCLE
C   REPLI  NUMBER OF SIMULATED PERIODS
C   SATDMD SATISFIED DEMAND
C   SDCOS  ESTIMATED STANDARD DEVIATION OF AVERAGE COST
C          PER PERIOD
```

```
C   SDMEAN SDCOS AS A FRACTION OF THE MEAN ESTIMATE
C   SEED   SEED FOR RANDOM NUMBER GENERATION
C   SHEXAC EXACT FRACTION OF PERIODS IN WHICH SHORTAGES
C          SHOULD OCCUR
C   SHSIMU FRACTION OF PERIODS IN WHICH SHORTAGES
C          OCCURED DURING SIMULAT
C   SUM    ACCUMULATES TO EXPECTED NUMBER OF STOCKOUTS
C   U      ONE RANDOM SAMPLE VALUE FROM UNIFORM (0,1)
C   Y      INITIAL QUANTITY AVAILABLE TO SATISFY DEMAND
C
C   DECLARATIONS:
      INTEGER REPLI,N,SEED,ISEED,INTVL1,INTVL2,SATDMD,Y,
     *        N1,N2,DMD,DMD1(1000),DMD2(1000),ITMQIT,CTR
      REAL U(1),P1,P2,PI1,PI2,COSCRT,COSCUM,COSSQ,AVECOS,
     *     SDCOS,APPROX,MU,MU1,MU2,BIAELI,SUM,SHSIMU,
     *     SDMEAN,DEVCDF,DEVMON,DEVHYP
      REAL*8 PMF(1000),PMF1(100 ),PMF2(100 ),COSHYP,COSCDF,
     *       SHEXAC
C   INITIALIZATION:
      N      =   20
      N1     =    4
      N2     =    5
      P1     =  0.25
      P2     =  0.75
      PI1    =  100.0
      PI2    = 1000.0
      Y      =      7
      REPLI  = 100
      SEED   = 150449
      INTVL1 = N/N1
      INTVL2 = N/N2
C
      ISEED  = SEED
      COSCUM = 0.0
      COSSQ  = 0.0
      CTR = 0
C
C   EACH EXECUTION OF THE FOLLOWING DO-LOOP GENERATES ONE
C   VALUE FOR THE 'SHORTAGE COST PER PERIOD' BY SIMULATING
C   ONE BERNOULLI-TRIAL FOR EACH REPAIR-CYCLE
C     (I.E. INTVL1, INTVL2 ) OF THE END ITEMS
C
      DO 300 I=1,REPLI
         J = 1
         WHILE ( J.LE.N )
            DMD1(J)  = 0
            DMD2(J)  = 0
            J = J + 1
         ENDWHILE
         DMD = 0
         DO 100 J=1,N,INTVL1
            CALL SRND (SEED,U,1,2,0)
            IF (U(1).LE.P1)  THEN
               DMD1(J) = 1
               DMD = DMD + 1
            ENDIF
 100     CONTINUE
         DO 200 J=1,N,INTVL2
            CALL SRND (SEED,U,1,2,0)
            IF (U(1).LE.P2)  THEN
               DMD2(J) = 1
               DMD = DMD + 1
            ENDIF
 200     CONTINUE
```

```
C
C   THE PROGRAM ASSUMES THAT THE DEMAND OCCURS RIGHT AT THE
C   BEGINNING OF THE REPAIR CYCLE. THE VARIABLE 'BIAELI'
C   COMPENSATES THE RESULTING BIAS. THE BIAS OCCURS IN
C   SITUATIONS IN WHICH THERE IS ONLY ONE ITEM LEFT TO
C   SATISFY DEMAND AND THERE ARE TWO DEMANDS OCCURRING
C   IN THAT TIME UNIT. IN THE PROGRAM ALWAYS THE SAME
C   DEMAND SOURCE GETS SERVED FIRST.
C
C   THE ASSUMPTION THAT THE DEMAND OCCURS RIGHT AT THE BE-
C   GINNING OF THE REPAIR CYCLE IS OF MORE IMPORTANCE FOR
C   THE NUMERICAL RESULT OF THE SIMULATION THE SMALLER AND
C   THE MORE DIFFERENT THE NUMBERS OF REPAIR CYCLES ARE.
C
         BIAELI = 0.0
         IF ((DMD.GT.Y).AND.(Y.GT.0))  THEN
            SATDMD = 0
            J = 1
            LOOP
               IF ( DMD1(J).EQ.1 ) THEN
                  SATDMD = SATDMD + 1
                  DMD1(J)  = 0
                  ITMQIT   = 1
               ENDIF
               IF ( SATDMD.EQ.Y ) QUIT
               IF ( DMD2(J).EQ.1 ) THEN
                  SATDMD = SATDMD + 1
                  DMD2(J)  = 0
                  ITMQIT   = 2
               ENDIF
               IF ( SATDMD.EQ.Y ) QUIT
               J = J + 1
            ENDLOOP
            IF (( DMD2(J).EQ.1 ).AND.(ITMQIT.EQ.1))  THEN
               BIAELI =  (PI1 + PI2)/2.0 - PI2
            ENDIF
         ENDIF
         COSCRT = BIAELI
         IF ( DMD .GT.Y )  THEN
            CTR = CTR + 1
            J = 1
            WHILE (J.LE.N)
               COSCRT = COSCRT + FLOAT(DMD1(J))*PI1
     *                  + FLOAT(DMD2(J))*PI2
               J = J + 1
            ENDWHILE
         ENDIF
C
C   'COSCUM' AND 'COSSQ' CUMULATE THE VALUES WHICH ARE LATER
C   ON NEEDED FOR COMPUTATION OF THE SAMPLE MEAN AND
C   ESTIMATED STANDARD DEVIATION OF THE SAMPLE MEAN.
C
         COSCUM = COSCUM +COSCRT
         COSSQ  = COSSQ + COSCRT**2
 300  CONTINUE
```

```
C
C
      MU1 = P1*FLOAT(N1)
      MU2 = P2*FLOAT(N2)
      MU = MU1 + MU2
C
C   COMPUTATION OF SAMPLE MEAN AS MEAN ESTIMATE AND
C   COMPUTATION OF ESTIMATED STANDARD DEVIATION OF
C   MEAN ESTIMATE
C
      AVECOS = COSCUM/FLOAT(REPLI)
      AUX = COSCUM**2/FLOAT(REPLI)
      AUX1 = FLOAT(REPLI)*FLOAT(REPLI-1)
      SDCOS=SQRT((COSSQ-AUX)/AUX1)
      WRITE (9,601) REPLI,N,ISEED,N1,P1,MU1,PI1,Y,N2,P2,
     *              MU2,PI2
 601  FORMAT(' ',71('*'),/,' REPLICATIONS:',I6,T24,
     *  'TIME UNITS PER PERIOD:',I4,T53,'INIT-SEED=',I10,//,
     *  ' # OF REPCYCLS_1:',I3,'    P1=',F6.4,'    MU1=',F7.2,
     *  '    PI1=',
     *   F7.2,T67,'Y=',I4,/,' # OF REPCYCLS_2:',I3,
     *  '    P2=',F6.4,'    MU2=',F7.2,'    PI2=',F7.2,/ )
      WRITE (9,602) AVECOS,SDCOS
 602  FORMAT(' ','RESULT OF BERNOULLI TRIALS FOR EACH',
     *           ' REPAIR CYCLE:'/,
     *           ' ESTIMATED MEAN OF SHORTAGE COSTS PER',
     *           ' PERIOD',F8.3,/,
     *' ESTIMATED  ST-DEV. FOR ESTIMATE OF THE MEAN:',F8.3)
```

```
C   APPLYING THE MONOVARIABLE APPROXIMATING FORMULA AND
C   EVALUATING THE EXACT EXPECTED FRACTION OF PERIODS
C   IN WHICH SHORTAGES OCCUR
      I = 0
      WHILE ( I.LE.N1 )
         PMF1(I+1)=DGAMMA(DFLOAT(N1+1))/
     *              (DGAMMA(DFLOAT(N1-I+1))*
     *              DGAMMA(DFLOAT(I+1)))
         PMF1(I+1)  =  PMF1(I+1)*(DBLE(P1))**I *
     *                 (DBLE(1.0 - P1))**(N1-I)
         I = I+1
      ENDWHILE
      I = 0
      WHILE ( I.LE.N2 )
         PMF2(I+1)=DGAMMA(DFLOAT(N2+1))/
     *              (DGAMMA(DFLOAT(N2-I+1))*
     *              DGAMMA(DFLOAT(I+1)))
         PMF2(I+1)  =  PMF2(I+1)*(DBLE(P2))**I *
     *                 (DBLE(1.0 - P2))**(N2-I)
         I = I+1
      ENDWHILE
      I = 0
      WHILE ( I.LE.N1+N2 )
         PMF(I+1)  =  0.0D0
         I = I  +  1
      ENDWHILE
      I = 0
      WHILE ( I .LE.N1 )
         J = 0
         WHILE  ( J .LE.  N2 )
            PMF(I+J+1)=PMF(I+J+1) + PMF1(I+1)  *  PMF2(J+1)
            J = J  +  1
         ENDWHILE
         I = I  +  1
      ENDWHILE
      SUM = 0.0
      SHEXAC  =  0.0D0
      I = Y + 1
      WHILE (I.LE.N1  + N2)
         SUM  = SUM  +  FLOAT(I-Y)*SNGL(PMF(I  +  1))
         SHEXAC  = SHEXAC  +  PMF(I+1)
         I = I+1
      ENDWHILE
      APPROX=((PI1*MU1+PI2*MU2)/(MU1+MU2))*SUM
      WRITE  (9,604)  APPROX
 604  FORMAT (' ',/,'     RESULT OF MONOVAR. ',
     *              'APPROXIMATING FORMULA:',F8.3,/)
C
C
```

```
C
C
      CALL HYPGEO (N1,N2,PMF1,PMF2,PI1,PI2,Y,COSHYP)
C
C
      CALL SUBCDF (N1,N2,PMF1,PMF2,PI1,PI2,Y,COSCDF)
C
C
C   COMPUTATION OF SUMMARY STATISTICS
C
      SHEXAC = SHEXAC*100.0
      SHSIMU =  (CTR/FLOAT(REPLI))*100.0
      IF (AVECOS.GT.0.005)   THEN
         DEVCDF=  ( COSCDF /AVECOS - 1.0)*100.0
         DEVHYP=  ( COSHYP /AVECOS - 1.0)*100.0
         DEVMON=  (APPROX/AVECOS - 1.0)*100.0
         SDMEAN =  (SDCOS/AVECOS)*100.0
         WRITE (9,605) MU,SHEXAC,SHSIMU,SDMEAN,DEVMON,
     *                DEVHYP,DEVCDF
605      FORMAT (' SUMMARY:        MU=',F5.2,'        SHEXAC=',
     *           F6.2,'%        SHSIMU=',F6.2,'%',//,
     *           ' SD/MEAN=',F6.1,'%      DEVMON=',F6.1,'%',
     *       T39,'DEVHYP=',F6.1,T53,'%    DEVCDF=',F6.1,'%')
      ELSE
         WRITE (9,606) MU,SHEXAC,SHSIMU
606      FORMAT (' SUMMARY:        MU=',F5.2,'        SHEXAC=',
     *           F6.2,'%        SHSIMU=',F6.2,'%',//,
     *           ' SD/MEAN= *******    DEVMON= ********',T39,
     *           'DEVHYP= ********',T57,'DEVCDF= ********')
      ENDIF
      STOP
      END
```

```
C
      SUBROUTINE HYPGEO (X1MAX,X2MAX,PMF1,PMF2,PI1,PI2,
     *                   Y,COST)
C
C   THIS SUBROUTINE HAS AS INPUT PARAMETERS:
C   X1MAX MAXIMAL VALUE DEMAND SOURCE#1
C   X2MAX MAXIMAL VALUE DEMAND SOURCE#2
C   PMF1  PROBAB.MASS FCT FOR DEMAND SOURCE#1 ,
C         ALWAYS HAS TO START AT ZERO (I.E. 0,1,2, ...)
C   PMF2  PROBAB.MASS FCT FOR DEMAND SOURCE#2 ,
C         ALWAYS HAS TO START AT ZERO (I.E. 0,1,2, ...)
C   PI1   SHORTAGE COST ITEM #1
C   PI2   SHORTAGE COST ITEM #2
C   Y     PURCHASED QUANTITY
C   COST  RETURNS VALUE OF HYPERGEOM. APPROXIMATION TO
C         MAIN PROGRAM
C
C   HYPGEO EVALUATES THE EXPECTED VALUE OF SHORTAGE
C   COSTS ACCORDING TO THE FORMULA FOR THE EXPECTED VALUE
C   OF THE HYPERGEOMETRIC DISTRIBUTION FOR EACH POSSIBLE
C   COMBINATION OF X1 AND X2
C
C   THE SUBROUTINE OUTPUTS AN APPROXIMATE VALUE FOR THE
C   EXPECTED SHORTAGE COST PER PERIOD
C
      INTEGER X1MAX,X2MAX,SHORT,Y
      REAL PI1,PI2
      REAL*8 PMF1(100 ),PMF2(100 ),COST
      COST = 0.0D0
      I1 = 0
      WHILE (I1.LE. X1MAX)
         I2 = 0
         WHILE (I2.LE. X2MAX)
            IF (I1+I2 .GT. Y) THEN
               SHORT = I1 + I2 - Y
               COST  = COST + PMF1(I1+1)*PMF2(I2+1) *
     *                 (DFLOAT(SHORT*I1)*DBLE(PI1)
     *                 DFLOAT(SHORT*I2)*DBLE(PI2))/
     *                 DFLOAT(I1+I2)
            ENDIF
            I2 = I2 + 1
         ENDWHILE
         I1 = I1 + 1
      ENDWHILE
      WRITE (9,601) COST
 601  FORMAT ('   RESULT OF HYPERGEO. APPROXIMATING',
     *        ' FORMULA:',F8.3,/)
      RETURN
      END
C
```

```
      SUBROUTINE SUBCDF (X1MAX,X2MAX,PMF1,PMF2,PI1,PI2,
C    *                  Y,COSCDF)
C   THIS SUBROUTINE HAS AS INPUT PARAMETERS:
C   X1MAX MAXIMAL VALUE DEMAND SOURCE#1
C   X2MAX MAXIMAL VALUE DEMAND SOURCE#2
C   PMF1  PROBAB.MASS FCT FOR DEMAND SOURCE#1 ,
C         ALWAYS HAS TO START AT ZERO (I.E. 0,1,2, ...)
C   PMF2  PROBAB.MASS FCT FOR DEMAND SOURCE#2 ,
C         ALWAYS HAS TO START AT ZERO (I.E. 0,1,2, ...)
C   PI1   SHORTAGE COST ITEM #1
C   PI2   SHORTAGE COST ITEM #2
C   SHORT SHORTAGE #OF ITEMS
C   Y     PURCHASED QUANTITY
C
C   COSCDF RETURNS VALUE OF CDF ORIENTATED APPROXIMATION
C   TO MAIN PROGRAM
C   THE SUBROUTINE OUTPUTS AN APPROXIMATE VALUE FOR THE
C   EXPECTED SHORTAGE COST PER PERIOD
C
C   SUBROUTINE INTERN VARIABLES:
C   CDF1  CUMULATIVE PROBAB. DISTRIBUTION  DEMAND SOURCE#1,
C   CDF2  CUMULATIVE PROBAB. DISTRIBUTION  DEMAND SOURCE#2,
C   Y1    ASSUMED SATISFIED DMD FOR END ITEM #1,
C         TRUNCATED TO INTEGER
C   Y2    ASSUMED SATISFIED DMD FOR END ITEM #2,
C         TRUNCATED TO INTEGER
C   W1    WEIGHT FOR SHORTAGE COST PI1
C   W2    WEIGHT FOR SHORTAGE COST PI2
C
      INTEGER X1MAX,X2MAX,SHORT,Y,Y1,Y2
      REAL PI1,PI2
      REAL*8 PMF1(100 ),PMF2(100 ),CDF1(100 ),CDF2(100 ),
     *       COSCDF,W1,W2
      CDF1(0+1)  = PMF1(0+1)
      DO 100 I=1,X1MAX
         CDF1(I+1)  = CDF1(I)  + PMF1(I+1)
 100  CONTINUE
      CDF2(0+1)  = PMF2(0+1)
      DO 200 I=1,X2MAX
         CDF2(I+1)  = CDF2(I)  + PMF2(I+1)
 200  CONTINUE
      COSCDF =  0.0D0
      I1 = 0
      WHILE (I1.LE. X1MAX)
         I2 = 0
         WHILE  (I2.LE. X2MAX)
            IF  (I1+I2 .GT.  Y)  THEN
               SHORT = I1 + I2 - Y
               Y1 =INT((FLOAT(I1)/FLOAT(I1+I2))*FLOAT(Y))
               W1=(  (CDF1(I1+1)-CDF1(Y1+1))/CDF1(I1+1)  )
     *             *DFLOAT(I1)
               Y2 =INT((FLOAT(I2)/FLOAT(I1+I2))*FLOAT(Y))
               W2=(  (CDF2(I2+1)-CDF2(Y2+1))/CDF2(I2+1)  )
     *             *DFLOAT(I2)
               COSCDF  = COSCDF + PMF1(I1+1)*PMF2(I2+1) *
     *                   DFLOAT(SHORT)*
     *                   ( (W1*PI1 + W2*PI2) / (W1+W2) )
            ENDIF
            I2 = I2 + 1
         ENDWHILE
         I1 = I1 + 1
      ENDWHILE
      WRITE (9,601) COSCDF
 601  FORMAT ('          RESULT OF CDF APPROXIMATING',
     *        ' FORMULA:',F8.3,/)
      RETURN
      END
$ENTRY
$END
```

APPENDIX B

COMPUTER PROGRAM FOR SIMULATING N BINOMIAL DEMAND SOURCES

```
$JOB
C                                NAME: WILHELM F. NOEGGERATH
C                                      DATE: JULY, 21ST 1984
C
C      BINOMIAL DISTRIBUTION, N DEMAND SOURCES
C         - SIMULATICN OF BERNOULLI TRIALS PER REPAIR CYCLE
C         - APPLYING MONOVARIABLE APPROXIMATION FORMULA
C           BY USING NORMAL APPROXIMATION TO BINOMIAL DIST'S
C
C   VARIABLES:
C   .AUX.  AUXILIARY VARIABLES
C   AVECOS AVERAGE COST PER PERIOD
C   COSCRT COST FOR CURRENT SIMULATED PERIOD
C   COSCUM CUMULATED COST FOR ALL SIMULATED PERIODS
C   COSSQ  CUMULATED PERIOD SQUARED COSTS
C   CRTDMD VECTOR WHICH STORES ORIGINAL DEMAND OF CURRENT
C          TIME UNIT
C   CUMDMD CUMULATES DEMAND DURING PERIOD
C   CTR    COUNTS PERIODS IN WHICH SHORTAGES OCCUR DURING
C          SIMULATION
C   DEVMON DEVIATION OF RESULT OF MONOVARIABLE APPROXIMATION
C          FOR EXPECTED SHORTAGE COST PER PERIOD FROM
C          APPROXIMATION RESULT IN PERCENT
C   DEVNSH DEVIATION OF RESULT OF MONOVARIABLE APPROXIMATION
C          FOR EXPECTED NUMBER OF SHORTAGES PER PERIOD FROM
C          SIMULATION RESULT IN PERCENT
C   DEVPIA DEVIATION OF THE AVERAGE OF THE UNIT SHORTAGE
C          COST AS USED IN THE NORMAL APPROXIMATION FROM
C          THE AVERAGE UNIT SHORTAGE COST AS OBSERVED
C          DURING SIMULATION
C   DMD    DEMAND MATRIX FOR ONE PERIOD
C          (ROW=DMD-SOURCE, COL=TIME-UNIT)
C   INTVL  VECTOR WITH LENGTH OF REPAIR CYCLES FOR
C          DIFFERENT DMD SOURCES
C   N      NUMBER OF DISCRETE TIME UNITS PER PERIOD
C  ***  N MUST BE A MULTIPLE OF EACH ELEMENT OF REPCYC    ***
C   NOFDMD NUMBER OF DIFFERENT DEMAND SOURCES
C   NOFSHN AVERAGE NUMBER OF SHORTAGES PER PERIOD
C          DERIVED FROM NORMAL APPROXIMATION
C   NOFSHS AVERAGE NUMBER OF SHORTAGES PER PERIOD
C          OBSERVED DURING SIMULATION
C   P      VECTOR , PROBABILITIES OF DEMAND
C          IN ONE REPAIR CYCLE
C   PI     VECTOR , UNIT SHORTAGE COSTS FOR DIFFERENT
C          DEMAND SOURCES
C   PINORA MEAN WEIGHTED AVERAGE UNIT SHORTAGE COST
C   PISIMA AVERAGE UNIT SHORTAGE COST AS OBSERVED DURING
C          SIMULATION
C   REPCYC VECTOR WITH #OF REPAIR CYCLES PER PERIOD
C   REPLI  NUMBER OF SIMULATED PERIODS
C   SATDMD CUMULATES SATISFIED DEMAND FOR CURRENT PERIOD
C   SDAVE  ESTIMATED STANDARD DEVIATION FOR THE MEAN
C          ESTIMATE FOR THE EXPECTED SHORTAGE COST PER
C          PERIOD DERIVED FROM SIMULATION DIVIDED BY THE
C          ESTIMATE I.E. DIVIDED BY THE SAMPLE MEAN
```

```
C     SDCOS  ESTIMATED STANDARD DEVIATION OF SIMULATED
C            AVERAGE SHORTAGE COST PER PERIOD
C     SDNOSH ESTIMATED STAND.-DEVIATION OF
C            NUMBER SHORTAGES PER PERIOD
C     SEED   SEED FOR RANDOM NUMBER GENERATION
C     SHCRT  #OF SHORTAGES FOR CURRENT SIMULATED PERIOD
C     SHCUM  CUMULATED #OF SHORTAGES FOR ALL SIMULATED PERIODS
C     SHSIMU FRACTION OF PERIODS IN WHICH SHORTAGES
C            OCCURED DURING SIMULATION
C     SHSQ   CUMULATED PERIOD SQUARED #OF SHORTAGES
C     U      ONE RANDOM SAMPLE VALUE FROM UNIFORM (0,1)
C     UNTDMD CUMULATES DEMAND FOR ONE TIME UNIT
C     Y      INITIAL QUANTITY AVAILABLE TO SATISFY DEMAND
C
C     DECLARATIONS:
        INTEGER CTR,CRTDMD(50),INTVL(50),N,NOFDMD,REPCYC(50),
       *        REPLI,SATDMD,SEED,UNTDMD,UNTSAT,XMAX,Y,CUMDMD
        REAL    AVECOS,COSCRT,COSCUM,COSSQ,DMD(50,100),P(50),
       *        PI(50),SDCOS,SDNOSH,SHCRT,SHCUM,SHSIMU,SHSQ,
       *        MU,U(1),NOFSHN,NOFSHS,APPROX,PINORA,SDAVE,
       *        PISIMA,DEVMON,DEVNSH,DEVPIA
C
C     WITH THE CURRENT DIMENSION THE PROGRAM IS ABLE TO HANDLE
C     50 DEMAND DISTRIBUTIONS FOR A PERIOD OF 100 TIME UNITS.
C     FOR SETTINGS OF MORE THAN 50 DISTRIBUTIONS OR MORE THAN
C     100 TIME UNITS THE DIMENSION STATEMENTS HAVE TO BE
C     CHANGED
C
        SEED    =     255
        READ (5,10) NOFDMD,N,Y,REPLI
   10   FORMAT (7X,I3,8X,I4,6X,I4,19X,I7)
        I=1
        WHILE (I.LE.NOFDMD)
           READ (5,20) REPCYC(I),P(I),PI(I)
   20      FORMAT (21X,I3,9X,F6.4,13X,F8.2)
           I=I+1
        ENDWHILE
        WRITE (9,30) NOFDMD,N,REPLI,SEED
   30   FORMAT (' ','NOFDMD=',I2,'   #OF TIME UNITS=',I3,
       *        '   REPLI=',I5,'   I-SEED=',I8,//,
       *        T2,'#OF REP-',T13,'PROBA-',T38,'UNIT-SHORT.',
       *        T55,'FIT',/,T2,'CYCLES',T13,'BILITY',T25,
       *        'MEAN',T38,'COST',T53,'N*P',T58,'N*Q',/)
        I=1
        WHILE (I.LE.NOFDMD)
           AUX  = FLOAT(REPCYC(I))*P(I)
           AUX1 = FLOAT(REPCYC(I))*(1.0-P(I))
           WRITE (9,40) I,REPCYC(I),I,P(I),I,AUX,I,PI(I),
       *                AUX,AUX1
   40      FORMAT (' ',I2,':',I3,I6,': ',F4.2,I6,': ',
       *                F6.2,I5,': ',F7.2, 3X,F5.1,F5.1)
           INTVL(I) = N/REPCYC(I)
           I=I+1
        ENDWHILE
C
        SUMMU  = 0.0
        SUMVAR = 0.0
        XMAX   =  0
        DO  50 I=1,NOFDMD
           MU = FLOAT(REPCYC(I))*P(I)
           SUMMU = SUMMU + MU
           SUMVAR = SUMVAR + MU*(1.0-P(I))
           XMAX = XMAX + REPCYC(I)
   50   CONTINUE
        STDDEV = SQRT(SUMVAR)
        WRITE (9,60) SUMMU,STDDEV,Y,XMAX
   60   FORMAT (' ',/,'TOTAL MEAN=',F6.2,'   TOTAL S-DEV=',
       *              F5.2,'   Y=',I3,'    MAX-DMD=',I4)
C
```

```
      COSCUM = 0.0
      COSSQ  = 0.0
      SHCUM  = 0.0
      SHSQ   = 0.0
      CTR = 0
C
C   EACH EXECUTION OF THE FOLLOWING DO-LOOP GENERATES ONE
C   VALUE FOR THE 'SHORTAGE COST PER PERIOD' BY SIMULATING
C   ONE BERNOULLI-TRIAL FOR EACH REPAIR-CYCLE (I.E.INTVL(I))
C   OT THE END-ITEMS
C
      DO 900 I=1,REPLI
         DO 200 J=1,NOFDMD
            DO 100 K=1,N
               DMD(J,K) = 0.0
  100       CONTINUE
  200    CONTINUE
         CUMDMD = 0
         DO 400 J=1,NOFDMD
            IAUX = INTVL(J)
            DO 300 K=1,N,IAUX
               CALL SRND (SEED,U,1,2,0)
               IF ( U(1).LE.P(J)) THEN
                  DMD(J,K) = 1.0
                  CUMDMD = CUMDMD + 1
               ENDIF
  300       CONTINUE
  400    CONTINUE
         IF ((CUMDMD.GT.Y).AND.(Y.GT.0))   THEN
            SATDMD = 0
            K=1
            LOOP
               UNTDMD = 0
               DO 500 L=1,NOFDMD
                  IF ( DMD(L,K).EQ.1.0 ) THEN
                     UNTDMD = UNTDMD + 1
                     CRTDMD(L) = 1
                  ELSE
                     CRTDMD(L) = 0
                  ENDIF
  500          CONTINUE
               DO 600 J=1,NOFDMD
                  IF (DMD(J,K).EQ.1.0)  THEN
                     SATDMD = SATDMD + 1
                     DMD(J,K) = 0.0
                  ENDIF
  600          CONTINUE
               IF (SATDMD.GE.Y) QUIT
               K = K+1
            ENDLOOP
C
C   IF MORE THAN THE AVAILABLE QUANTITY GOT ASSIGNED
C   TO THE CUSTOMERS A CORRECTION HAS TO BE PERFORMED.
C
C   ADDITIONALLY THE BIAS CAUSED BY CHECKING THE DEMAND
C   MATRIX ALWAYS IN THE SAME SEQUENCE HAS TO BE ELIMINATED.
C
            IF (SATDMD.GT.Y) THEN
               DO 750 J=1,NOFDMD
                  IF (CRTDMD(J).EQ.1) THEN
                     DMD(J,K) = FLOAT(SATDMD - Y)/
     *                          FLOAT(UNTDMD)
                  ENDIF
  750          CONTINUE
            ENDIF
         ENDIF
         COSCRT = 0.0
         SHCRT  = 0.0
         IF ( CUMDMD .GT. Y )  THEN
            CTR = CTR + 1
            DO 850 J=1,NOFDMD
               DO 800 K=1,N
                  COSCRT = COSCRT + DMD(J,K)*PI(J)
                  SHCRT = SHCRT + DMD(J,K)
  800          CONTINUE
  850       CONTINUE
         ENDIF
         COSCUM = COSCUM +COSCRT
         COSSQ  = COSSQ + COSCRT**2
         SHCUM  = SHCUM + SHCRT
         SHSQ   = SHSQ + SHCRT**2
  900 CONTINUE
C
C
      AVECOS = COSCUM/FLOAT(REPLI)
      AUX2 = COSCUM**2/FLOAT(REPLI)
      AUX3 = FLOAT(REPLI)*FLOAT(REPLI-1)
      SDCOS=SQRT((COSSQ-AUX2)/AUX3)
      NOFSHS = SHCUM/FLOAT(REPLI)
      AUX4 = SHCUM**2/FLOAT(REPLI)
      SDNOSH=SQRT((SHSQ-AUX4)/AUX3)
      SHSIMU= (FLOAT(CTR)/FLOAT(REPLI))*100.0
      WRITE (9,950) AVECOS,NOFSHS,SHSIMU,SDCOS,SDNOSH
  950 FORMAT(' ',/,'      SIMULATION: AVE-COST=',F8.2,
     *       '  #OF SHOR''S=',F9.4,/,'SHSIMU=',F6.2,
     *       '%      SD-COST=',F8.2,'  SD-#OF SH''S=',F8.4)
C
      CALL NORMAL(Y,NOFDMD,REPCYC,P,PI,NOFSHN,STDDEV,SUMMU,
     *            XMAX,PINORA,APPROX)
C
C   COMPUTATION OF SUMMARY STATISTICS
C
      IF (NOFSHS.GT.0.00005)     THEN
         PISIMA = AVECOS / NOFSHS
         IF ((AVECOS.GT.0.005).AND.(PISIMA.GT.0.005)) THEN
            SDAVE  =  (SDCOS / AVECOS)*100.0
            DEVMON =  (APPROX/AVECOS - 1.0)*100.0
            DEVNSH =  (NOFSHN/NOFSHS - 1.0)*100.0
            DEVPIA =  (PINORA/PISIMA - 1.0)*100.0
            WRITE (9,960) SDAVE,PISIMA,PINORA,DEVMON,DEVNSH,
     *                    DEVPIA
 960        FORMAT(' ',/,'SUMMARY STATISTICS:',/,
     *                T1,'SDAVE=',F9.2,'%',T20,'PISIMA=',
     *                F8.2,'$',T40,'PINORA=',F8.2,'$',/,T1,
     *                'DEVMON=',F8.2,'%',T20,'DEVNSH=',F8.2,
     *                '%',T40,'DEVPIA=',F8.2,'%')
         ELSE
            WRITE (9,970)
         ENDIF
      ELSE
         WRITE (9,970)
  970    FORMAT (' ',/,'NO SURVEY STATISTICS COMPUTED,',
     *            /,'AT LEAST ONE DIVISOR TOO SMALL')
      ENDIF
      STOP
      END
C
```

```
C
      SUBROUTINE NORMAL (Y,NOFDMD,REPCYC,P,PI,NOFSHN,STDDEV,
     *                   SUMMU,XMAX,PINORA,APPROX)
C
C   THE SUBROUTINE 'NORMAL' COMPUTES AN EXPECTED VALUE FOR
C   THE SHORTAGE COST PER PERIOD ACCORDING TO THE MONO-
C   VARIABLE APPROXIMATION FORMULA BY USING A NORMAL
C   DISTRIBUTION AS AN APPROXIMATION FOR THE DISTRIBUTION
C   OF THE SUM OF DEMANDS PER PERIOD.
C
C
C   SUBROUTINE INTERN VARIABLES:
C   CDFL   STANDARD NORMAL DISTRIBUTION EVALUATED AT LL
C   CDFR   STANDARD NORMAL DISTRIBUTION EVALUATED AT RL
C   CRT    CURRENT SUM OF DEMANDS ( GREATER THAN Y )
C   LL     (CRT - 0.5) TRANSFORMED TO STANDARD NORMAL
C   RL     (CRT + 0.5) TRANSFORMED TO STANDARD NORMAL
C   SHORT  CURRENT NUMBER OF SHORTAGES
C   SUMPI  SCALAR, CUMULATED P(I)*PI(I); I=1, ...,N
C
C   DECLARATIONs
        INTEGER NOFDMD,Y,REPCYC(9),XMAX
        REAL APPROX,SUMMU,MU,SUMVAR,SUMPI,PI(50),P(50),
     *          SHORT,STDDEV,NOFSHN,Z,PINORA,CRT,
     *          CDFL,CDFR,LL,RL
C
        SUMPI = 0.0
        DO 1 I=1,NOFDMD
           SUMPI = SUMPI + FLOAT(REPCYC(I))*P(I)*PI(I)
    1   CONTINUE
        SHORT = 1.0
        CRT = FLOAT(Y+1)
        NOFSHN = 0.0
        LL = (CRT - 0.5 -SUMMU)/STDDEV
C
C   THE ISML LIBRARY SUBROUTINE 'MDNOR( A,B )' EVALUATES
C   THE STANDARD NORMAL DISTRIBUTION AT A AND passes ITS
C   VALUE VIA THE PARAMETER B TO THE CALLING PROGRAM
C
        CALL MDNOR ( LL, CDFL)
        WHILE ( CRT.LE.FLOAT(XMAX) ) DO
           RL = (CRT + 0.5 - SUMMU)/STDDEV
           CALL MDNOR ( RL, CDFR)
           NOFSHN = NOFSHN + SHORT * (CDFR - CDFL)
           CDFL = CDFR
           SHORT = SHORT + 1.0
           CRT = CRT + 1.0
        ENDWHILE
        PINORA = SUMPI/SUMMU
        APPROX = PINORA * NOFSHN
        WRITE (9,2) APPROX,NOFSHN
    2   FORMAT (' ',/,'NORMAL-APPROXIM.:',
     *           ' AVE-COST=',F8.2,'  #OF SHOR''S=',F9.4)
      RETURN
      END
$ENTRY
```

APPENDIX C

SIMULATION RESULTS FOR THREE DEMAND SOURCES, Y INCREASING

```
------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 72  REPLI=20000  I-SEED=  901941

 #OF REP-    PROBA-                      UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN           COST             N*P  N*Q
  1: 18      1: 0.20      1:    3.60     1:   300.00      3.6 14.4
  2: 36      2: 0.40      2:   14.40     2:   300.00     14.4 21.6
  3: 12      3: 0.30      3:    3.60     3:   100.00      3.6  8.4
TOTAL MEAN= 21.60   TOTAL S-DEV= 3.75   Y= 20   MAX-DMD=  66

      SIMULATION: AVE-COST=  655.11  #OF SHOR'S=   2.4178
SHSIMU= 61.06%     SD-COST=    5.27  SD-#OF SH'S=  0.0195

NORMAL-APPROXIM.: AVE-COST=  646.67  #OF SHOR'S=   2.4250

SUMMARY STATISTICS:
SD/AVE=     0.80%   PISIMA=  270.95$    PINORA=  266.67$
DEVMON=    -1.29%   DEVNSH=    0.30%    DEVPIA=   -1.58%
------------------------------------------------------------------

------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 72  REPLI=20000  I-SEED=  901941

 #OF REP-    PROBA-                      UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN           COST             N*P  N*Q
  1: 18      1: 0.20      1:    3.60     1:   300.00      3.6 14.4
  2: 36      2: 0.40      2:   14.40     2:   300.00     14.4 21.6
  3: 12      3: 0.30      3:    3.60     3:   100.00      3.6  8.4
TOTAL MEAN= 21.60   TOTAL S-DEV= 3.75   Y= 23   MAX-DMD=  66

      SIMULATION: AVE-COST=  246.61  #OF SHOR'S=   0.9046
SHSIMU= 30.36%     SD-COST=    3.37  SD-#OF SH'S=  0.0124

NORMAL-APPROXIM.: AVE-COST=  238.36  #OF SHOR'S=   0.8938

SUMMARY STATISTICS:
SD/AVE=     1.36%   PISIMA=  272.63$    PINORA=  266.67$
DEVMON=    -3.35%   DEVNSH=   -1.19%    DEVPIA=   -2.19%
------------------------------------------------------------------

------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 72  REPLI=20000  I-SEED=  901941

 #OF REP-    PROBA-                      UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN           COST             N*P  N*Q
  1: 18      1: 0.20      1:    3.60     1:   300.00      3.6 14.4
  2: 36      2: 0.40      2:   14.40     2:   300.00     14.4 21.6
  3: 12      3: 0.30      3:    3.60     3:   100.00      3.6  8.4
TOTAL MEAN= 21.60   TOTAL S-DEV= 3.75   Y= 26   MAX-DMD=  66

      SIMULATION: AVE-COST=   63.14  #OF SHOR'S=   0.2293
SHSIMU=  9.80%     SD-COST=    1.66  SD-#OF SH'S=  0.0061

NORMAL-APPROXIM.: AVE-COST=   58.49  #OF SHOR'S=   0.2193

SUMMARY STATISTICS:
SD/AVE=     2.64%   PISIMA=  275.29$    PINORA=  266.67$
DEVMON=    -7.36%   DEVNSH=   -4.37%    DEVPIA=   -3.13%
------------------------------------------------------------------
```

APPENDIX D

SIMULATION RESULTS FOR THREE DEMAND SOURCES.

```
-------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 60  REPLI=20000  I-SEED=     1809

 #OF REP-    PROBA-                        UNIT-SHORT.       FIT
 CYCLES      BILITY       MEAN             COST            N*P  N*Q
  1: 10      1: 0.20      1:     2.00      1:  100.00      2.0  8.0
  2: 12      2: 0.30      2:     3.60      2:  300.00      3.6  8.4
  3: 15      3: 0.80      3:    12.00      3:  200.00     12.0  3.0
TOTAL MEAN= 17.60   TOTAL S-DEV= 2.55    Y= 20    MAX-DMD=  37

      SIMULATION: AVE-COST=    49.11  #OF SHOR'S=    0.2333
SHSIMU= 12.61%      SD-COST=    1.09  SD-#OF SH'S=  0.0051

NORMAL-APPROXIM.: AVE-COST=    48.93  #OF SHOR'S=    0.2340

SUMMARY STATISTICS:
SD/AVE=     2.22%   PISIMA=  210.47$    PINORA=  209.09$
DEVMON=    -0.37%   DEVNSH=     0.29%   DEVPIA=    -0.65%
-------------------------------------------------------------------
```

```
-------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 72  REPLI=20000  I-SEED=    84009

 #OF REP-    PROBA-                        UNIT-SHORT.       FIT
 CYCLES      BILITY       MEAN             COST            N*P  N*Q
  1: 12      1: 0.40      1:     4.80      1:  300.00      4.8  7.2
  2: 24      2: 0.50      2:    12.00      2:  300.00     12.0 12.0
  3: 18      3: 0.60      3:    10.80      3:  100.00     10.8  7.2
TOTAL MEAN= 27.60   TOTAL S-DEV= 3.63    Y= 32    MAX-DMD=  54

      SIMULATION: AVE-COST=    45.55  #OF SHOR'S=    0.2010
SHSIMU=  9.05%      SD-COST=    1.24  SD-#OF SH'S=  0.0054

NORMAL-APPROXIM.: AVE-COST=    43.69  #OF SHOR'S=    0.1970

SUMMARY STATISTICS:
SD/AVE=     2.72%   PISIMA=  226.57$    PINORA=  221.74$
DEVMON=    -4.08%   DEVNSH=    -1.99%   DEVPIA=    -2.13%
-------------------------------------------------------------------
```

```
-------------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 60  REPLI=20000  I-SEED=      891

 #OF REP-    PROBA-                        UNIT-SHORT.       FIT
 CYCLES      BILITY       MEAN             COST            N*P  N*Q
  1: 15      1: 0.70      1:    10.50      1:  100.00     10.5  4.5
  2: 20      2: 0.30      2:     6.00      2:  300.00      6.0 14.0
  3: 12      3: 0.60      3:     7.20      3:  300.00      7.2  4.8
TOTAL MEAN= 23.70   TOTAL S-DEV= 3.20    Y= 27    MAX-DMD=  47

      SIMULATION: AVE-COST=    49.95  #OF SHOR'S=    0.2356
SHSIMU= 11.55%      SD-COST=    1.22  SD-#OF SH'S=  0.0055

NORMAL-APPROXIM.: AVE-COST=    52.36  #OF SHOR'S=    0.2477

SUMMARY STATISTICS:
SD/AVE=     2.44%   PISIMA=  211.97$    PINORA=  211.39$
DEVMON=     4.83%   DEVNSH=     5.11%   DEVPIA=    -0.27%
-------------------------------------------------------------------
```

```
--------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 60  REPLI=20000  I-SEED=  312774

 #OF REP-    PROBA-                     UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN           COST            N*P  N*Q

  1: 10      1: 0.30     1:    3.00     1:  300.00       3.0  7.0
  2: 15      2: 0.60     2:    9.00     2:  300.00       9.0  6.0
  3: 20      3: 0.50     3:   10.00     3:  100.00      10.0 10.0

TOTAL MEAN= 22.00   TOTAL S-DEV= 3.27   Y= 27   MAX-DMD=  45

      SIMULATION: AVE-COST=    17.29   #OF SHOR'S=   0.0904
SHSIMU=  4.88%     SD-COST=     0.68   SD-#OF SH'S=  0.0033

NORMAL-APPROXIM.: AVE-COST=    18.45   #OF SHOR'S=   0.0882

SUMMARY STATISTICS:
SD/AVE=     3.93%   PISIMA=  191.12$    PINORA=  209.09$
DEVMON=     6.71%   DEVNSH=   -2.46%    DEVPIA=    9.40%
--------------------------------------------------------------

--------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 88  REPLI=20000  I-SEED=  932675

 #OF REP-    PROBA-                     UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN           COST            N*P  N*Q

  1: 11      1: 0.10     1:    1.10     1:  100.00       1.1  9.9
  2: 44      2: 0.30     2:   13.20     2:  100.00      13.2 30.8
  3: 22      3: 0.90     3:   19.80     3:  300.00      19.8  2.2

TOTAL MEAN= 34.10   TOTAL S-DEV= 3.49   Y= 39   MAX-DMD=  77

      SIMULATION: AVE-COST=    24.60   #OF SHOR'S=   0.1268
SHSIMU=  5.86%     SD-COST=     0.85   SD-#OF SH'S=  0.0043

NORMAL-APPROXIM.: AVE-COST=    27.17   #OF SHOR'S=   0.1257

SUMMARY STATISTICS:
SD/AVE=     3.47%   PISIMA=  193.93$    PINORA=  216.13$
DEVMON=    10.45%   DEVNSH=   -0.89%    DEVPIA=   11.45%
--------------------------------------------------------------

--------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 84  REPLI=20000  I-SEED=  928698

 #OF REP-    PROBA-                     UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN           COST            N*P  N*Q

  1: 21      1: 0.30     1:    6.30     1:  300.00       6.3 14.7
  2: 42      2: 0.60     2:   25.20     2:  300.00      25.2 16.8
  3: 14      3: 0.20     3:    2.80     3:  100.00       2.8 11.2

TOTAL MEAN= 34.30   TOTAL S-DEV= 4.09   Y= 40   MAX-DMD=  77

      SIMULATION: AVE-COST=    39.55   #OF SHOR'S=   0.1366
SHSIMU=  6.12%     SD-COST=     1.31   SD-#OF SH'S=  0.0046

NORMAL-APPROXIM.: AVE-COST=    42.72   #OF SHOR'S=   0.1506

SUMMARY STATISTICS:
SD/AVE=     3.32%   PISIMA=  289.46$    PINORA=  283.67$
DEVMON=     8.00%   DEVNSH=   10.20%    DEVPIA=   -2.00%
--------------------------------------------------------------
```

```
---------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 48   REPLI=20000   I-SEED=    3311

 #OF REP-    PROBA-                      UNIT-SHORT.          FIT
 CYCLES      BILITY        MEAN          COST               N*P   N*Q

   1: 12     1: 0.80     1:     9.60    1:   100.00         9.6   2.4
   2: 24     2: 0.40     2:     9.60    2:   250.00         9.6  14.4
   3: 16     3: 0.10     3:     1.60    3:   250.00         1.6  14.4

TOTAL MEAN= 20.80   TOTAL S-DEV= 3.02   Y= 24   MAX-DMD=  52

      SIMULATION: AVE-COST=    43.50   #OF SHOR'S=    0.2196
SHSIMU= 10.85%      SD-COST=    1.05   SD-#OF SH'S=   0.0053

NORMAL-APPROXIM.: AVE-COST=    39.98   #OF SHOR'S=    0.2212

SUMMARY STATISTICS:
SD/AVE=     2.42%   PISIMA=  198.10$    PINORA=  180.77$
DEVMON=    -8.09%   DEVNSH=    0.72%    DEVPIA=   -8.75%
---------------------------------------------------------------
```

```
---------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 80   REPLI=20000   I-SEED=  395113

 #OF REP-    PROBA-                      UNIT-SHORT.          FIT
 CYCLES      BILITY        MEAN          COST               N*P   N*Q

   1: 20     1: 0.90     1:    18.00    1:   100.00        18.0   2.0
   2: 80     2: 0.30     2:    24.00    2:   100.00        24.0  56.0
   3: 40     3: 0.90     3:    36.00    3:   300.00        36.0   4.0

TOTAL MEAN= 78.00   TOTAL S-DEV= 4.71   Y= 82   MAX-DMD= 140

      SIMULATION: AVE-COST=   102.12   #OF SHOR'S=    0.5370
SHSIMU= 17.13%      SD-COST=    1.98   SD-#OF SH'S=   0.0105

NORMAL-APPROXIM.: AVE-COST=    99.36   #OF SHOR'S=    0.5167

SUMMARY STATISTICS:
SD/AVE=     1.94%   PISIMA=  190.17$    PINORA=  192.31$
DEVMON=    -2.70%   DEVNSH=   -3.78%    DEVPIA=    1.12%
---------------------------------------------------------------
```

```
---------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 60   REPLI=20000   I-SEED= 7655599

 #OF REP-    PROBA-                      UNIT-SHORT.          FIT
 CYCLES      BILITY        MEAN          COST               N*P   N*Q

   1: 10     1: 0.20     1:     2.00    1:   100.00         2.0   8.0
   2: 30     2: 0.40     2:    12.00    2:   100.00        12.0  18.0
   3: 15     3: 0.20     3:     3.00    3:   300.00         3.0  12.0

TOTAL MEAN= 17.00   TOTAL S-DEV= 3.35   Y= 22   MAX-DMD=  55

      SIMULATION: AVE-COST=    14.71   #OF SHOR'S=    0.1114
SHSIMU=  5.48%      SD-COST=    0.55   SD-#OF SH'S=   0.0039

NORMAL-APPROXIM.: AVE-COST=    13.23   #OF SHOR'S=    0.0978

SUMMARY STATISTICS:
SD/AVE=     3.72%   PISIMA=  131.97$    PINORA=  135.29$
DEVMON=   -10.03%   DEVNSH=  -12.24%    DEVPIA=    2.52%
---------------------------------------------------------------
```

```
-------------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 60   REPLI=20000   I-SEED=    34876

 #OF REP-    PROBA-                   UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN        COST             N*P  N*Q

  1: 20      1: 0.20      1:    4.00   1:   300.00      4.0 16.0
  2: 30      2: 0.30      2:    9.00   2:   300.00      9.0 21.0
  3: 15      3: 0.60      3:    9.00   3:   100.00      9.0  6.0

TOTAL MEAN= 22.00    TOTAL S-DEV= 3.62    Y= 26    MAX-DMD=  65

      SIMULATION: AVE-COST=    57.84  #OF SHOR'S=    0.2477
SHSIMU= 10.92%      SD-COST=     1.44  SD-#OF SH'S=   0.0061

NORMAL-APPROXIM.: AVE-COST=    53.10  #OF SHOR'S=    0.2434

SUMMARY STATISTICS:
SD/AVE=     2.49%   PISIMA=  233.45$    PINORA=  218.18$
DEVMON=    -8.20%   DEVNSH=    -1.77%    DEVPIA=    -6.54%
-------------------------------------------------------------------

-------------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 72   REPLI=20000   I-SEED=   176532

 #OF REP-    PROBA-                   UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN        COST             N*P  N*Q

  1: 12      1: 0.90      1:   10.80   1:   100.00     10.8  1.2
  2: 36      2: 0.40      2:   14.40   2:   100.00     14.4 21.6
  3: 18      3: 0.50      3:    9.00   3:   300.00      9.0  9.0

TOTAL MEAN= 34.20    TOTAL S-DEV= 3.77    Y= 39    MAX-DMD=  66

      SIMULATION: AVE-COST=    26.99  #OF SHOR'S=    0.1781
SHSIMU=  8.02%      SD-COST=     0.80  SD-#OF SH'S=   0.0051

NORMAL-APPROXIM.: AVE-COST=    27.45  #OF SHOR'S=    0.1799

SUMMARY STATISTICS:
SD/AVE=     2.96%   PISIMA=  151.55$    PINORA=  152.63$
DEVMON=     1.71%   DEVNSH=     0.99%    DEVPIA=     0.71%
-------------------------------------------------------------------

-------------------------------------------------------------------
NOFDMD= 3   #OF TIME UNITS= 78   REPLI=20000   I-SEED= 3480610

 #OF REP-    PROBA-                   UNIT-SHORT.        FIT
 CYCLES      BILITY       MEAN        COST             N*P  N*Q

  1: 13      1: 0.40      1:    5.20   1:   100.00      5.2  7.8
  2: 78      2: 0.20      2:   15.60   2:   100.00     15.6 62.4
  3: 39      3: 0.70      3:   27.30   3:   300.00     27.3 11.7

TOTAL MEAN= 48.10    TOTAL S-DEV= 4.88    Y= 55    MAX-DMD= 130

      SIMULATION: AVE-COST=    37.89  #OF SHOR'S=    0.1828
SHSIMU=  6.73%      SD-COST=     1.23  SD-#OF SH'S=   0.0059

NORMAL-APPROXIM.: AVE-COST=    36.71  #OF SHOR'S=    0.1719

SUMMARY STATISTICS:
SD/AVE=     3.25%   PISIMA=  207.27$    PINORA=  213.51$
DEVMON=    -3.12%   DEVNSH=    -5.96%    DEVPIA=     3.01%
-------------------------------------------------------------------
```

```
-----------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 66  REPLI=20000  I-SEED=  394861

 #OF REP-    PROBA-                    UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN          COST            N*P  N*Q

  1: 11      1: 0.05     1:    0.55    1:  300.00      0.5 10.4
  2: 66      2: 0.30     2:   19.80    2:  300.00     19.8 46.2
  3: 33      3: 0.10     3:    3.30    3:  100.00      3.3 29.7

TOTAL MEAN= 23.65   TOTAL S-DEV= 4.17   Y= 30   MAX-DMD= 110

      SIMULATION: AVE-COST=    35.25  #OF SHOR'S=    0.1292
SHSIMU=  5.46%     SD-COST=     1.26  SD-#OF SH'S=   0.0046

NORMAL-APPROXIM.: AVE-COST=    31.08  #OF SHOR'S=    0.1142

SUMMARY STATISTICS:
SD/AVE=     3.57%   PISIMA=  272.73$    PINORA=  272.09$
DEVMON=   -11.84%   DEVNSH=  -11.63%    DEVPIA=    -0.23%
-----------------------------------------------------------------

-----------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 72  REPLI=20000  I-SEED=  901941

 #OF REP-    PROBA-                    UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN          COST            N*P  N*Q

  1: 18      1: 0.20     1:    3.60    1:  300.00      3.6 14.4
  2: 36      2: 0.40     2:   14.40    2:  300.00     14.4 21.6
  3: 12      3: 0.30     3:    3.60    3:  100.00      3.6  8.4

TOTAL MEAN= 21.60   TOTAL S-DEV= 3.75   Y= 23   MAX-DMD=  66

      SIMULATION: AVE-COST=   246.61  #OF SHOR'S=    0.9046
SHSIMU= 30.36%     SD-COST=     3.37  SD-#OF SH'S=   0.0124

NORMAL-APPROXIM.: AVE-COST=   238.36  #OF SHOR'S=    0.8938

SUMMARY STATISTICS:
SD/AVE=     1.36%   PISIMA=  272.63$    PINORA=  266.67$
DEVMON=    -3.35%   DEVNSH=   -1.19%    DEVPIA=    -2.19%
-----------------------------------------------------------------

-----------------------------------------------------------------
NOFDMD= 3  #OF TIME UNITS= 40  REPLI=20000  I-SEED=   32467

 #OF REP-    PROBA-                    UNIT-SHORT.       FIT
 CYCLES      BILITY      MEAN          COST            N*P  N*Q

  1: 10      1: 0.30     1:    3.00    1:  100.00      3.0  7.0
  2: 20      2: 0.60     2:   12.00    2:  100.00     12.0  8.0
  3: 40      3: 0.50     3:   20.00    3:  300.00     20.0 20.0

TOTAL MEAN= 35.00   TOTAL S-DEV= 4.11   Y= 38   MAX-DMD=  70

      SIMULATION: AVE-COST=   122.37  #OF SHOR'S=    0.5349
SHSIMU= 19.58%     SD-COST=     2.16  SD-#OF SH'S=   0.0096

NORMAL-APPROXIM.: AVE-COST=   118.98  #OF SHOR'S=    0.5552

SUMMARY STATISTICS:
SD/AVE=     1.77%   PISIMA=  228.78$    PINORA=  214.29$
DEVMON=    -2.77%   DEVNSH=    3.81%    DEVPIA=    -6.34%
-----------------------------------------------------------------
```

APPENDIX E

SIMULATION RESULTS FOR FIVE DEMAND SOURCES

```
------------------------------------------------------------------------
NOFDMD= 5  #OF TIME UNITS= 60  REPLI=20000  I-SEED=  243078

 #OF REP-     PROBA-                      UNIT-SHORT.          FIT
 CYCLES       BILITY      MEAN            COST               N*P  N*Q

  1:  10      1: 0.10     1:    1.00      1:   200.00        1.0   9.0
  2:  30      2: 0.30     2:    9.00      2:   300.00        9.0  21.0
  3:  20      3: 0.50     3:   10.00      3:   100.00       10.0  10.0
  4:  12      4: 0.70     4:    8.40      4:   300.00        8.4   3.6
  5:  15      5: 0.90     5:   13.50      5:   200.00       13.5   1.5

TOTAL MEAN= 41.90   TOTAL S-DEV= 4.01   Y= 42   MAX-DMD=  87

       SIMULATION: AVE-COST=  333.30   #OF SHOR'S=   1.5449
SHSIMU= 43.77%      SD-COST=    3.62   SD-#OF SH'S=  0.0165

NORMAL-APPROXIM.: AVE-COST=  336.42   #OF SHOR'S=   1.5456

SUMMARY STATISTICS:
SD/AVE=     1.09%   PISIMA=  215.74$    PINORA=  217.66$
DEVMON=     0.93%   DEVNSH=    0.04%    DEVPIA=    0.89%
------------------------------------------------------------------------
```

```
------------------------------------------------------------------------
NOFDMD= 5  #OF TIME UNITS= 60  REPLI=20000  I-SEED=  243078

 #OF REP-     PROBA-                      UNIT-SHORT.          FIT
 CYCLES       BILITY      MEAN            COST               N*P  N*Q

  1:  10      1: 0.10     1:    1.00      1:   200.00        1.0   9.0
  2:  30      2: 0.30     2:    9.00      2:   300.00        9.0  21.0
  3:  20      3: 0.50     3:   10.00      3:   100.00       10.0  10.0
  4:  12      4: 0.70     4:    8.40      4:   300.00        8.4   3.6
  5:  15      5: 0.90     5:   13.50      5:   200.00       13.5   1.5

TOTAL MEAN= 41.90   TOTAL S-DEV= 4.01   Y= 46   MAX-DMD=  87

       SIMULATION: AVE-COST=   69.44   #OF SHOR'S=   0.3228
SHSIMU= 12.76%      SD-COST=    1.62   SD-#OF SH'S=  0.0074

NORMAL-APPROXIM.: AVE-COST=   69.07   #OF SHOR'S=   0.3173

SUMMARY STATISTICS:
SD/AVE=     2.33%   PISIMA=  215.13$    PINORA=  217.66$
DEVMON=    -0.54%   DEVNSH=   -1.70%    DEVPIA=    1.18%
------------------------------------------------------------------------
```

```
NOFDMD= 5  #OF TIME UNITS= 60  REPLI=20000  I=SEED=  243078

 #OF REP-    PROBA-                      UNIT-SHORT.          FIT
 CYCLES      BILITY       MEAN           COST              N*P   N*Q

  1:  10     1: 0.10      1:     1.00    1:   200.00       1.0   9.0
  2:  30     2: 0.30      2:     9.00    2:   300.00       9.0  21.0
  3:  20     3: 0.50      3:    10.00    3:   100.00      10.0  10.0
  4:  12     4: 0.70      4:     8.40    4:   300.00       8.4   3.6
  5:  15     5: 0.90      5:    13.50    5:   200.00      13.5   1.5

TOTAL MEAN= 41.90   TOTAL S-DEV= 4.01   Y= 48   MAX-DMD=  87

      SIMULATION: AVE-COST=    24.09   #OF SHOR'S=   0.1130
SHSIMU=  5.08%     SD-COST=     0.92   SD-#OF SH'S=  0.0042

NORMAL-APPROXIM.: AVE-COST=    24.05   #OF SHOR'S=   0.1105

SUMMARY STATISTICS:
SD/AVE=    3.82%   PISIMA=  213.08$    PINORA=  217.66$
DEVMON=   -0.15%   DEVNSH=   -2.25%    DEVPIA=    2.15%
--------------------------------------------------------------
```

APPENDIX F

OUTPUT TABLEAU OF THE SIMULATION PROGRAM FOR TWO DEMAND SOURCES

```
*************************************************************
REPLIC'NS:100000[1]  TIME UNITS PER PERIOD:20[2]  I-SEED=150449[3]

#OF REPCYCLS(1):4[4]    P1=0.25[5] MU1=1.00[6] PI1= 100.00[7] Y=7[8]
#OF REPCYCLS(2):5[9]    P2=0.75[10]MU2=3.75[11]PI2=1000.00[12]

RESULT OF BERNOULLI TRIALS FOR EACH REPAIR CYCLE:
ESTIMATED MEAN OF SHORTAGE COSTS PER PERIOD: 13.437[13]
ESTIMATED  ST-DEV. FOR ESTIMATE OF THE MEAN:  0.365[14]

    RESULT OF MONOVAR. APPROXIMATING FORMULA: 11.771[15]

   RESULT OF HYPERGEO. APPROXIMATING FORMULA:  9.332[16]

        RESULT OF CDF APPROXIMATING FORMULA: 13.439[17]

SUMMARY:[18]MU= 4.75[19]   SHEXAC=1.36%[20]     SHSIMU=  1.36%[21]

SD/MEAN=2.7%[22] DEVMON=-12.4% DEVHYP=-30.6 % DEVCDF=   0.0%[23]
*************************************************************
```

Comments referring to the output tableau:

1. Number of simulated periods;
2. Length of one period;
3. Initial seed, necessary to reproduce identical simulation result;
4. Number of repair cycles for end item no. 1 per period. For instance, if the time period has a length of 20 time units and the number of repair

cycles is 4 then each repair cycle has a length of 5 time units.

5. Probability of demand in one repair cycle for end item no.1;
6. Mean demand for period for end item no.1;
7. Unit shortage cost if one demand caused by end item no.1 not satisfied;
8. Purchased quantity available to satisfy period demand;
9. Number of repair cycles for end item no.2;
10. Probability of demand in one repair cycle for end item no.2;
11. Mean demand for period for end item no.2;
12. Unit shortage cost, one demand caused by end item no.2 not satisfied;
13. Sample mean of the shortage costs per period as observed during simulation;
14. Estimates precision of mean estimate.
15. Monovariable (McMasters') approximation formula;
16. Hypergeometric approximation formula;
17. CDF approximation formula;
18. The last two lines provide a condensed simulation summary.
19. Sum of expected demands from both demand sources for one period;
20. The program computes the distribution of the sum of the two binomial demand distributions and is therefore able to output an exact expected fraction of periods in which shortages occur. In this example 1360 periods with shortages are the expected number so the expected fraction is 1.36%.
21. Fraction of periods in which shortages occurred during simulation. When compared to SHEXAC it provides a numerical hint for the quality of the random numbers generated during the simulation.

22. The ratio of estimated standard deviation for the estimate of the mean to the sample mean. SD/MEAN decreases as the number of replications increases and the confidence interval for the mean derived from simulation becomes smaller. The value of SD/MEAN is the basis for the decision as to whether the number of replications ensures sufficient precision. The number of replications must be traded off against the desired precision.

23. DEVMON, DEVHYP, and DEVCDF express how far the three approximation formulas deviate from the simulation result. The complete last row allows a quick survey about the performance of the approximations. From the observed sample mean for the shortage costs per period and its estimated standard deviation, a confidence interval for the true unknown mean can be computed. It can be used to determine the value of the confidence level such that the confidence interval for the true mean would include the values of the approximation formulas. For instance if SD/MEAN = 5% and DEVMON = -10%, it follows that the value of the monovariable approximation formula is two estimated standard deviations away from the sample mean and would be covered by a confidence interval with a confidence level of at least .9545 i.e. $P(|Z| \leq 2) = .9545$.

LIST OF REFERENCES

1. McMasters, Alan W., *A Repair Parts Inventory Model for a Naval Air Rework Facility*, Naval Postgraduate School Report 54-80-04, Monterey, May 1980.

2. Slaybaugh, Ernest R., *A Preliminary Analysis of TF34-100/400 Jet Engine Rework Data in Support of the MRP System Implementation at NARF Alameda*, Master's Thesis, Naval Postgraduate School, Monterey, 1981.

3. Asselin, Andre S., *A Multi-Period Repair Parts Inventory Model for a Naval Air Rework Facility*, Master's Thesis, Naval Postgraduate School, Monterey, 1983.

4. Ross, Sheldon M., *Introduction to Probability Models*, second edition, Academic Press, 1980.

5. Feller, William, *An Introduction to Probability Theory and its Applications*, volume I, 2nd ed., John Wiley & Sons, Inc., 1962.

6. Lewis, Peter A.W. and Uribe, Luis, *The Naval Postgraduate School Random Number Package - LLRANDOMII*, Naval Postgraduate School Report 55-81-005, Monterey, February 1981.

7. Lewis, Peter A.W. *Simulation Lectures*, Handout distributed in Course OA3302 'System Simulation', Naval Postgraduate School, Monterey, Summer Quarter Academic Year 1983.

8. Conover, W.J., *Practical Nonparametric Statistics*, Wiley, 1980.

9. Olkin, I., Gleser, L.J., and Derman, C., *Probability Models and Applications*, Macmillan, 1980.

10. Miller, I. and Freund, J.E., *Probability and Statistics for Engineers*, 2nd ed., Prentice-Hall, 1977.

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